MA-125 6C,   CALCULUS I

Test 3, November 4, 2015

Name (Print last name first): ....

$$\boxed{\textbf{Show all your work and justify your answer!}}$$

$$\boxed{\textbf{No partial credit will be given for the answer only!}}$$

$$\boxed{\textbf{PART I}}$$

You must simplify your answer when possible.

All problems in Part I are 7 points each.

1. Evaluate $\displaystyle\int_0^{\pi/4} \sec^2(x)\, dx$.

2. Evaluate $\displaystyle\int x^2(x+1)^2\, dx$.

3. Evaluate $\int x^3 \sqrt[5]{x^4 + 3}\, dx$.

4. The average value of the function $f(x) = x^3$ on the interval $[0, 2]$.

5. Evaluate $\int \dfrac{x^3 + x}{\sqrt{x}}\, dx$.

6. Estimate $\int_1^4 \sin(x^3)\,dx$ using a Riemann sum with $n = 3$ terms and the **midpoint rule**. Do NOT provide numerical values, rather give an expression for the desired Riemann sum.

7. Evaluate $\int_{-5}^{5} \dfrac{\sin(x)}{x^4 + x^2 + 1}\,dx$.

8. State the first part of the Fundamental Theorem of Calculus in which the derivative of a certain integral is proven to be equal to the integrand (that is to the function which is being integrated). Use it to write AN EXPRESSION for the anti-derivative of the function $y = f(x) = \sin(x^3)$.

$$\boxed{\textbf{PART II}}$$

All problems in Part II are 11 points each.

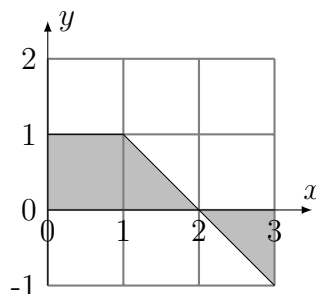1. Evaluate $\displaystyle\int \frac{x}{(2x+1)^{30}}\, dx.$

2. Evaluate $\displaystyle\int \frac{\sin(1/x)}{x^2}\, dx.$

3. Suppose the graph of a function $y = f(x)$ is shown in the plot below.
   (i) Find the value of its integral: $\int_0^3 f(x)\, dx$

   (ii) Let $g(x) = \int_0^x f(t)\, dt$. Is $g(x)$ increasing or decreasing on $(1, 2)$? [As always you must explain your answer!]

   The area of a triangle is $\frac{1}{2} \cdot$ base $\cdot$ height

4. If oil leaks out of a storage container at a rate of $(5t + 1)^{-2}$ $m^3/hour$ find the total amount which leaked from the container in the first 3 hours. Recall that the rate of change of an amount which changes with time equals the derivative of the function which reflects the amount in question.

SCRATCH PAPER

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