

MA 125-CT, CALCULUS I

September 22, 2015

Name (Print last name first):

TEST I

Show all your work! No partial credit will be given for the answer only!
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PART I

Part I consists of questions. Clearly write your answer in the space provided after each question. Show all of your your work!

All problems in Part I are 7 points each

Question 1

Use the **definition** of the derivative to show that the derivative of $y = f(x) = x^3 + x$ is $f'(x) = 3x^2 + 1$.

Question 2

Find the derivative of $f(x) = x^2 \cos(x)$

Answer:

Question 3

Find the derivative of $y = f(x) = \frac{1-x}{1+x}$.

Answer:

Question 4

Find the derivative of $y = f(x) = \sqrt[3]{x}(x + x^3)$.

Answer:

Question 5

Find the equation of the tangent line to the graph of $y = f(x) = \sin(x)$ at the point $a = \pi/3$.

Answer:

Question 6

Evaluate the limit $\lim_{x \rightarrow \infty} \frac{\sin(x^2)}{x^2}$

Answer:

Question 7

Evaluate the limit $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 9}$

Answer:

PART II

Part II consists of 4 problems. You must show correct reasons to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1 (12 points)

Suppose that $S(t) = t^5 - 5t$ is the position of a particle at time t (in seconds) on a line. Find:

- (a) the velocity at time t
- (b) the displacement from $t = 0$ to $t = 1$
- (c) the displacement from $t = 1$ to $t = 2$
- (d) the total distance traveled from $t = 0$ to $t = 2$.

Recall that the displacement could be positive or negative depending on the direction of movement.

Problem 2 (10 points)

Given the graph of the function $y = f(x)$ below find:

1. $\lim_{x \rightarrow -1^-} f(x) =$

2. $\lim_{x \rightarrow -1^+} f(x) =$

3. $\lim_{x \rightarrow -1} f(x) =$

4. $\lim_{x \rightarrow 2^-} f(x) =$

5. $\lim_{x \rightarrow 2^+} f(x) =$

6. $\lim_{x \rightarrow 2} f(x) =$

7. $\lim_{x \rightarrow \infty} f(x) =$

8. $\lim_{x \rightarrow -\infty} f(x) =$

9. State all intervals on which $f(x)$ is continuous.10. State all intervals where $f(x)$ is differentiable.

Problem 3 (10 points)

Find all points on the graph of $f(x) = x^3 + 2x$ where the tangent line is parallel to the line $y = 14x$.

Problem 5 (10 points)

Evaluate the following limits. Like always, justify your answers.

1. $\lim_{x \rightarrow \infty} \sqrt{x} + \sqrt{x+1}$

2. $\lim_{x \rightarrow \infty} \sqrt{x} - \sqrt{x+1}$

Scratch paper