PROGRAM WITH ABSTRACTS

Billiards and other dynamical systems

Dedicated to the memory of Kolya Chernov

Department of Mathematics, University of Alabama at Birmingham
March 15 - March 17, University Hall, UAB

WEDNESDAY, MARCH 15

All talks of the workshop will be held in room UH1005

Snacks will be provided between talks.

OPENING OF THE WORKSHOP: 1:15pm - 1:30pm

HONGKUN ZHANG, UNIVERSITY OF MASSACHUSETS

Title: *Convex billiards and their random perturbations*
Time: 1:30pm-2:30pm

Abstract: The study of chaotic billiards originated from Moscow, led by the fundamental work of Sinai, Bunimovich and Chernov. However, the study of hyperbolicity and entropy of convex billiards has been rather low, many open questions remain unsolved. I will go through some new convex billiards and review results about them, as well as review results about random perturbation of convex billiards.

MARK DEMERS, FAIRFIELD UNIVERSITY

Title: *Equilibrium states and measure of maximal entropy for finite horizon Sinai billiard flows.*
Time: 2:50pm-3:50pm

Abstract: We construct unique equilibrium states for a family of Holder continuous potentials for finite horizon Sinai billiard maps and then lift them to equilibrium states for the corresponding flows. This family includes the potentials corresponding to the measures of maximal entropy for both the billiard map and the billiard flow under a mild condition on recurrence to singularities. This condition is conjectured to hold for generic finite horizon Sinai billiard tables. The MMEs we construct are positive on all open sets and Bernoulli (which implies ergodic and mixing). This is joint work with Viviane Baladi and Jerome Carrand.

RENATO FERES, WASHINGTON UNIVERSITY IN ST.LOUIS
Time: 4:10pm-5:10pm

Title: *Chaotic lensed billiards*

Abstract: Lensed billiards are an extension of the notion of billiard dynamical systems obtained by adding a potential function in the form of a constant multiple of the indicator function of a subset of the billiard domain. This subset, called a lens, acts as a generalized scatterer, upon which

billiard trajectories either reflect or refract depending on their angle of incidence and the value of the potential constant. After reviewing the optical/mechanical analogy that motivates these billiard models, we explore how their dynamical properties, in particular Lyapunov exponents, depend on the potential parameter.

LEONID BUNIMOVICH, GEORGIA TECH
Time: 5:30pm-6:30pm

Title: *Wild Rose, Narcissus and other Elliptic Flowers Billiards*

Abstract: I will talk about a new class of billiards which demonstrates very unusual (and unexpected) behavior, which hopefully will help us understand better Hamiltonian dynamics.

THURSDAY, MARCH 16

HONGKUN ZHANG, UNIVERSITY OF MASSACHUSETS

Title: *Deep learning of chaotic systems I*
Time: 9am-10am

Abstract: I will start with the feed-forward Neural Networks, and the motivations of understanding neural networks from a mathematical point of view. Some mathematical theories behind machine learning, especially the learning of dynamical systems using neural networks, will be reviewed. For many complex real-world phenomena, only partial knowledge of their dynamics is known. DeepLearning (DL) provides efficient alternatives to learn dynamics from massive datasets. It achieves so by directly predicting the input-output mapping and bypassing numerical integration. Recent works have shown that DL can generate realistic predictions and significantly accelerate the simulation of physical dynamics relative to numerical solvers, from turbulence modeling to weather prediction. However, DL of chaotic systems have been rather difficult, because of the existence of positive Lyapunov exponents. I will present my recent results on DL of the Lorenz System, as well as the Standard Map, by constructing a new DL method based on the Long-short term memory (LSTM) neural network structures.

ALEXANDER BLUMENTHAL, GEORGIA TECH

Title: *Lyapunov exponents of high-dimensional, weakly dissipated SDE with applications to stochastic Galerkin-Navier-Stokes and Lorenz 96*
Time: 10:30am-11:30am

Abstract: In spite of decades of hard work and theoretical developments, it remains a major open challenge is to estimate the Lyapunov exponents of systems of practical interest on positive-area subsets of phase space. This problem is notoriously challenging, even for low-dimensional such as the one-parameter family of Chirikov standard maps, a model of the stretch-and-fold mechanism underlying mixing in real-world fluid flow for which numerical evidence suggests a positive Lyapunov exponent. Remarkably, in the presence of stochastic driving, estimating Lyapunov exponents becomes remarkably more tractable, and one can prove results on chaos far beyond the scope of what is currently possible for deterministic systems. This talk will discuss recent results, joint with J Bedrossian (UMD) and

S Punshon-Smith (IAS / Tulane), on establishing positive Lyapunov exponents for weakly dissipated stochastic differential equations, with applications to the Lorenz 96 model (a prototype of chaotic behavior in high dimensions) and Galerkin truncations of stochastic Navier-Stokes equations (the equations of motion for incompressible fluids). These results are based on an apparently new identity for Lyapunov exponents in terms of a partial Sobolev regularity of the stationary statistics for tangent directions.

LUNCH BREAK FROM 11:30am UNTIL 1:30pm

ALEXANDER GRIGO, OKLAHOMA UNIVERSITY (JOINT WITH LEONID BUNIMOVICH, GEORGIA TECH))

Title: *Existence of stable orbits in smooth stadium billiards*
Time: 1:30pm-2:30pm

Abstract: In this talk I will present joint work with L.Bunimovich addressing the interplay between smoothness, integrability and ergodicity for convex billiards. Specifically we will consider smooth stadium like billiards, and prove a number of results on the existence of stable periodic orbits as soon as the boundary of the billiard table is of class C^2. A key observation behind our results is the fact that as soon as the smoothness of the boundary is C^2 the resulting focusing boundary components fail to be absolutely focusing. The latter is the key ingredient in the standard construction of chaotic billiards with focusing boundary components.

NANDOR SIMANYI, UAB

Title: *Almost Every 1D Falling Ball System Is Ergodic*
Time: 3pm-4pm

Abstract: In the 1980s M. P. Wojtkowski introduced an interesting dynamical system 1D balls moving in a vertical half-line, colliding with each other and the hard floor elastically, and falling down under constant gravitation. To avoid the existence of linearly stable periodic orbits, one assumes that the masses of the particles are decreasing as we go up in the half line. He conjectured that all these systems are completely hyperbolic and ergodic.

Complete hyperbolicity of all such systems was shown in 1996. Here we describe a new algebraic approach to such systems that enable us to verify the conditions of the Local Ergodic Theorem for Dynamical Systems With Invariant Cone Fields (by Liverani and Wojtkowski) for almost all falling ball systems, thus proving their ergodicity.

FRIDAY, MARCH 17

HONGKUN ZHANG, UNIVERSITY OF MASSACHUSETS

Title *Deep learning of chaotic systems II*
Time: 8:30am-9:30am

Abstract: Deep learning has demonstrated great results on learning integrable Hamiltonian systems. However, for chaotic systems, or perturbed Hamiltonian systems, there has been very little work done. I will review the Hamiltonian Neural Network (HNN), as well as our newly constructed Perturbed Hamiltonian Neural Network (PHNN), and apply it to some dynamical systems. Machine learning of chaotic billiards is the ultimate goal of this research.

YAOFENG SU, GEORGIA TECH (JOINT WITH LEONID BUNIMOVICH, GEORGIA TECH)

Title: *Some new results of open dynamical systems*
Time: 9:50am-10:50am

Abstract: open dynamical systems describe hitting processes of orbits through a target in a phase space of a dynamical system. One of the main questions of open chaotic dynamical systems is to study a statistical property (called a Poisson limit) of the hitting process. I will present some new results in this area, applications include some dissipative systems and hyperbolic billiards. This is a joint work with Prof. Bunimovich.

FEDERICO BONETTO, GEORGIA TECH

Title: *Autonomous evolution for the speed of 2 electrons in a thermostated system*.
Time: 11:10am-12:10pm

Abstract: We study a system of two particles moving in 1 dimension under the influence of an electric field E and a deterministic thermostat that keeps the total kinetic energy constant. The particles undergo random elastic collisions with ”virtual” obstacles at a rate proportional to their speed. We show that in the small E/large time limit the evolution of the particles speed is described by a SDE uniformly in time up to the steady state. This is a joint work with Livia Corsi.