Assignment # 3 Due: Thursday, September 1
(Turn in for credit!)

Activities based on previous lecture:

1. A particle of mass $m$ moves in one dimension under a force $F(x) = \gamma x$, where $\gamma$ is a positive constant (i.e., a force linear on the position $x$).

   Calculate the work done by this force in the three different situations below.

   a. When particle moves in configuration space from point $a$ to point $b$ according to the following orbit:

   b. When particle moves in configuration space from point $a$ to point $b$ according to the following orbit:

   c. When particle moves in configuration space from point $a$ to point $b$ according to the following orbit:

   d. Compare the values you found for the work in situations (a), (b), (c) and discuss the meaning of your finding.
2. A particle of mass $m$ moves in one dimension under a force $F(v) = -\alpha v$, where $\alpha$ is a positive constant and $v$ is the particle velocity. Assume that at point $x=a$ the particle has an initial velocity $v_0$, and that $b \leq a + \frac{mv_0}{\alpha}$.

Calculate the work done by this force in the three different situations below.

a. Calculate the work done by this force when the particle moves in configuration space from point $a$ to point $b$ according to the following orbit:

![Diagram of particle movement from a to b](image)

b. Discuss the physical meaning of the result you found for the work in part (a). Your discussion should focus on the following aspects:

i. Showing that the work done by $F(v)$ does NOT depend only on the end points $a$ and $b$.

ii. Showing that the work done by $F(v)$ depends on how the particle moves from $a$ to $b$ (i.e., the work depends on the history, on the details of the orbit in configuration space, on the details of the path in one dimension, etc.).

c. Explain what happens with the work done by $F(v)$ in the following limiting cases:

i. When $b = a$. What is the physical meaning of this situation?

ii. When $b = a + \frac{mv_0}{\alpha}$. What is the physical meaning of this situation?
3. (20 pts) A particle of mass $m$ moves in one dimension under the force:

$$F(x) = 4 \ c \ (x^3 - x) \quad \text{where } c > 0$$

a) Discuss the effect of this force on the total mechanical energy of the particle. Is it appropriate to define a potential energy for the motion of this particle? Why?

b) Find an expression for the potential energy $V(x)$ of the particle (Choose a reference point such that any arbitrary constants vanish)

c) Draw by hand a sketch of the potential energy $V(x)$ (No computer use upfront, please!) *(You may check with a computer afterwards)*

d) For which values of the total energy will the motion be:
   - Bound (i.e., confined). Find the turning points.
   - Unbound, with change of direction. Find the turning points.
   - Unbound, with no change of direction.

