QMI Lesson 11:
Applications of the Second Derivative

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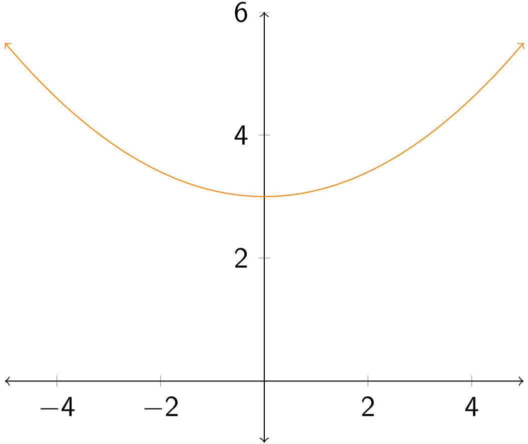
So, if the second derivative is positive, the slopes of the tangent lines are **increasing**. And if its negative, the slopes of the tangent lines are **decreasing**.

Visualizing the Second Derivative

Would this function have a positive or negative second derivative at the points drawn?



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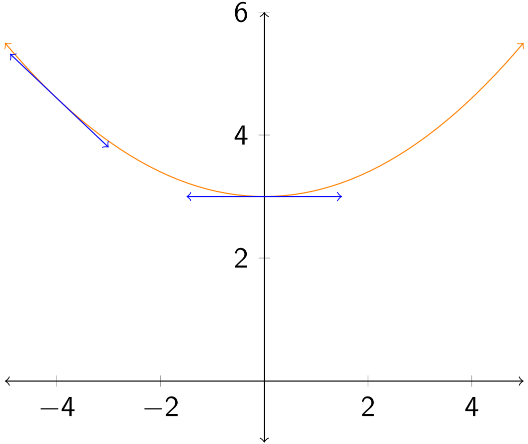

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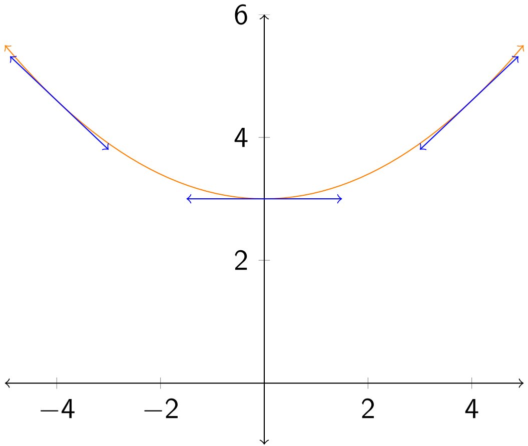

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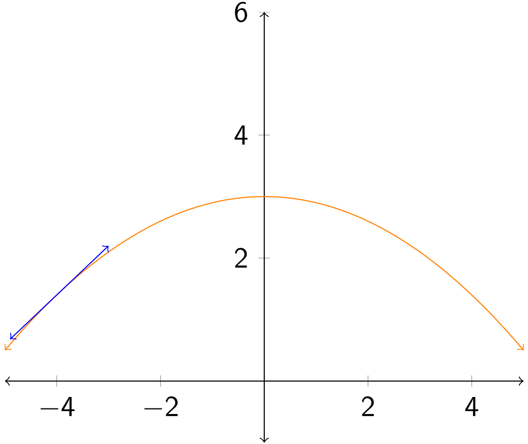

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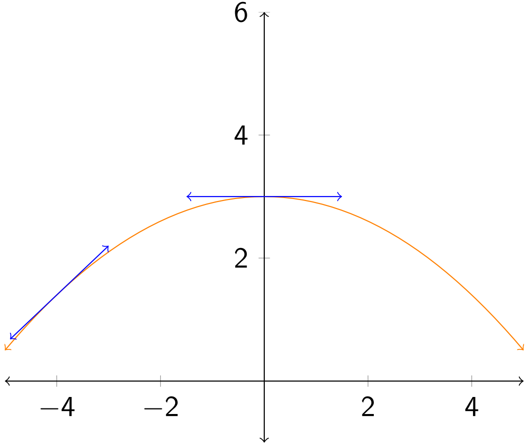

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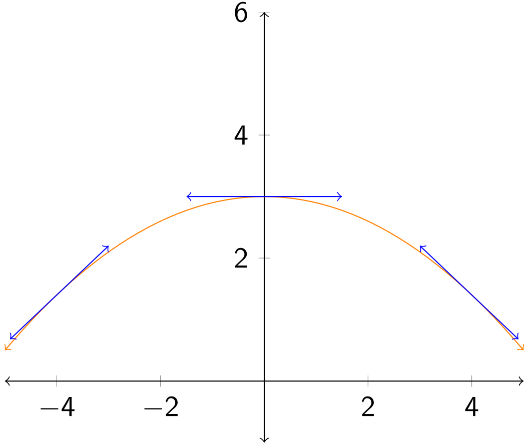

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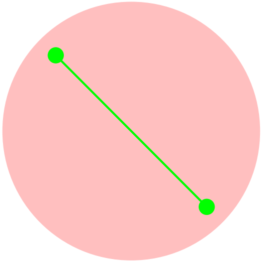
A figure (i.e. a shape) is convex if, given any two points x and y inside the figure,the line L connecting the figure ($L = tx + (1-t)y$) lies entirely in the figure.



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A differentiable function f is concave up on an interval (a, b) if f' is increasing on that interval.

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1. *If* $f''(x) > 0$ *for every* x *in* (a, b)*, then the graph of* f *is concave up on* (a, b)*.*
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Using the previous theorem, we can find the intervals on which a function is concave up/down by following these steps.

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1. Find all points where $f'' = 0$ or is undefined, and break the number line over these points.
2. Test the intervals. If f'' is positive in an interval, then f is concave up on the corresponding interval. If it's negative, then f is concave down on the corresponding interval.
3. If f is concave up on (a, b) and on (b, c), then it is concave up on (a, c).

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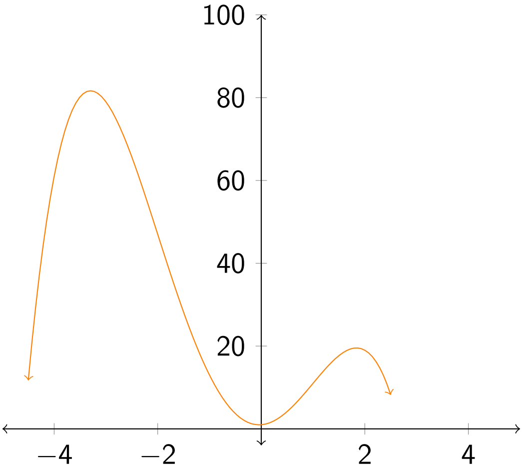
Thus $0 = f''(x) = -12(x^2 - x + 2) = -12(x-1)(x+2)$. Thus, $f''(x) = 0$ when $x = -2, 1$.

Interval	Test	Concavity
$(-\infty, -2)$	$f''(-3) < 0$	down
$(-2, 1)$	$f''(0) > 0$	up
$(1, \infty)$	$f''(3) < 0$	down

Concavity: Graph



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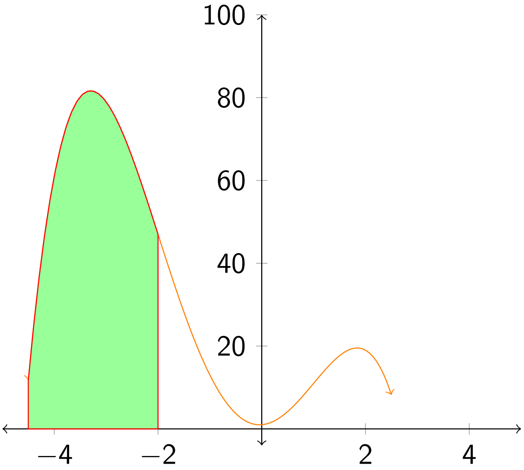

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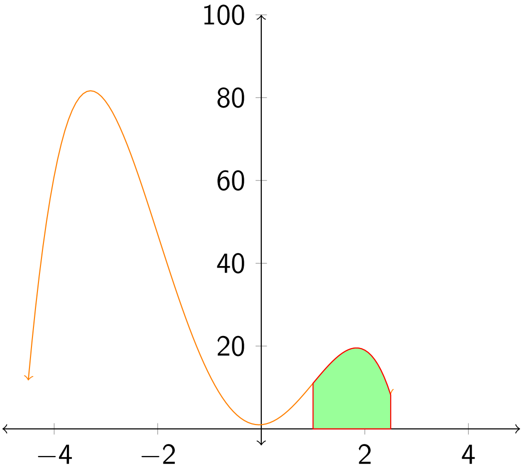

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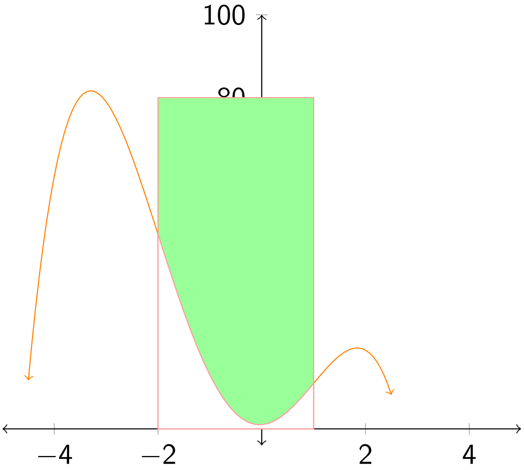

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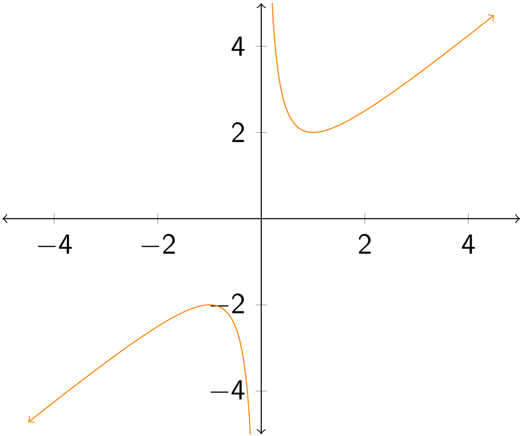

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Note: It's important that the tangent line exists!

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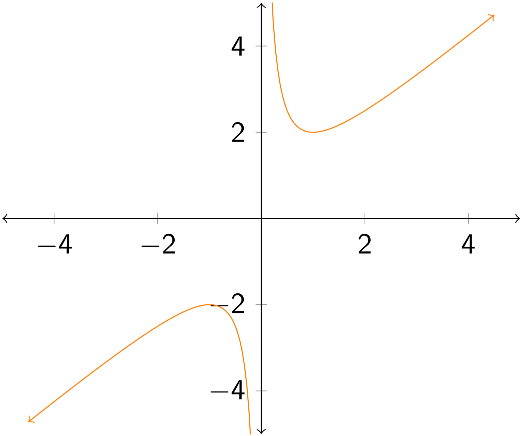

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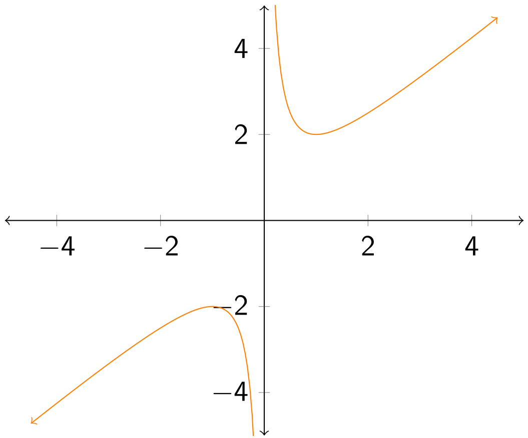

This function has no inflection points. Although the concavity changes as you pass over $x = 0$, the tangent line does not exist at $x = 0$.

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3. Determine the sign of f'' immediately to the left and right of each number c found in the previous step. If there is a change in sign of f as we move across $x = c$ and if the tangent line at $x = c$ exists, then $(c, f(c))$ is an inflection point of f.

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Therefore, $(-2, \frac{8}{3})$ and $(2, -\frac{8}{3})$ are inflection points.

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Therefore, $(-2, \frac{8}{3})$ and $(2, -\frac{8}{3})$ are inflection points. But $x = 0$ does not correspond to an inflection point because f has no tangent line at 0!

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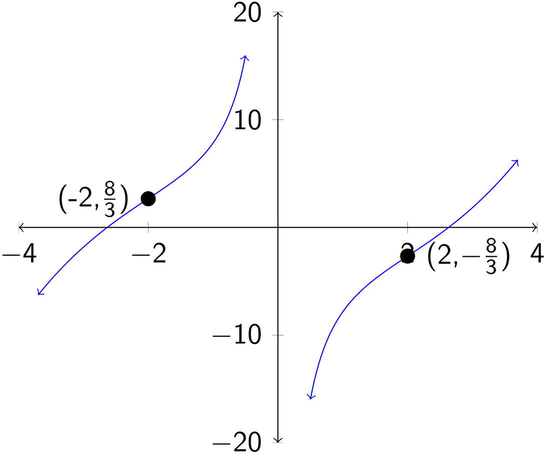

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2. A polynomial of degree 3 has exactly one inflection point. **True!** Why?

Graph Sketching

Sketch the graph of a function f where

- $f(-1) = -4$, $f(1) = -2$, $f(3) = 1$.
- $f'(x) > 0$ on $(-1, 3)$ and $f'(x) < 0$ on $(-\infty, -1) \cup (3, \infty)$.
- $f'(-1) = f''(1) = f'(3) = 0$.
- $f''(x) > 0$ on $(-\infty, 1)$ and $f''(x) < 0$ on $(1, \infty)$.

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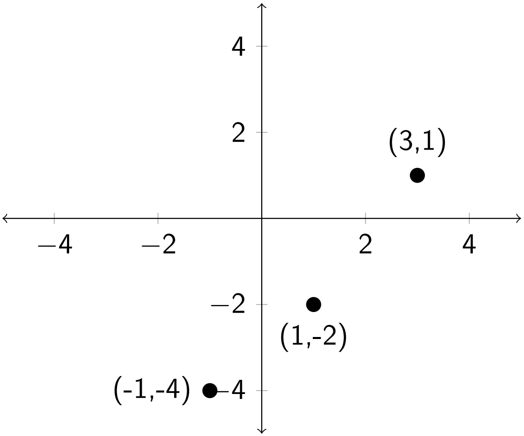

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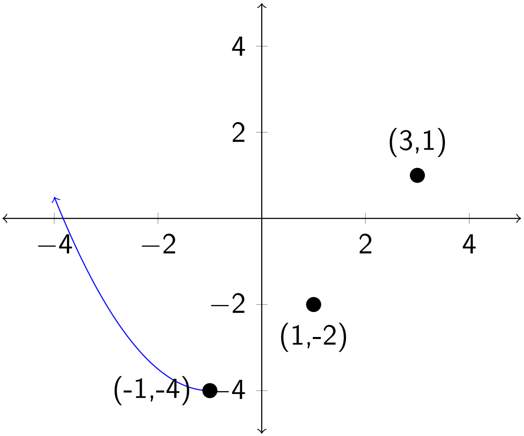

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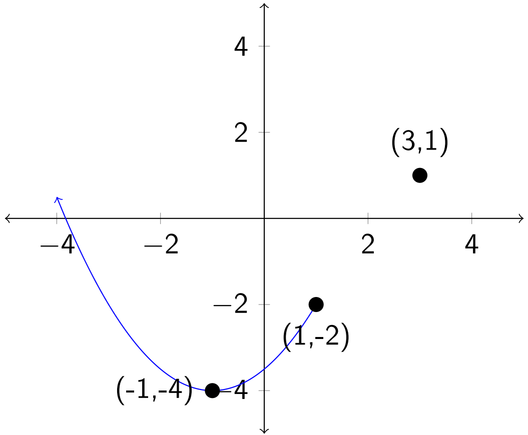

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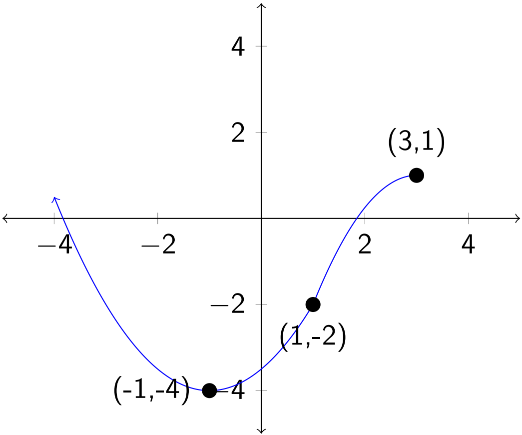

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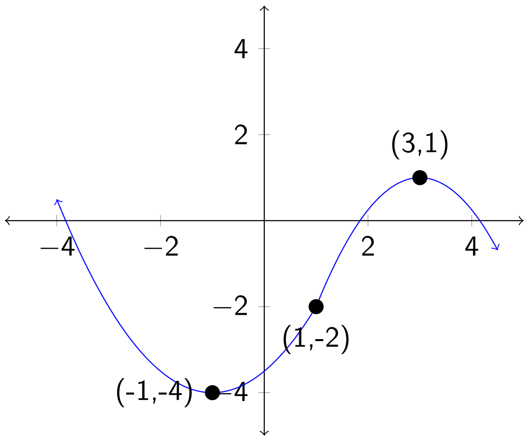

Example

An economy's consumer price index (CPI) is described by the function $f(t) = -0.2t^3 + 3t^2 + 100$ with $(0 \leq t \leq 10)$, where $t = 0$ corresponds to 2003. Find the point of inflection of the function f, and discuss its significance.

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Therefore, the rate of inflation (i.e. the rate of change of the rate of change of the CPI) is increasing between 2003 and 2008 and decreasing between 2008 and 2013.

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   - C If $f''(c) = 0$ or does not exist, then the test fails.

Example

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Assignment

Read 4.3-4.4. Do problems 6, 8, 12, 16, 26, 40, 60, 72, 80, 92, 108 in 4.2 (due October 6).