QMI Lesson 12:
Curve Sketching and the Extreme Value Theorem

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Curve Sketching: Steps

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6. Plot a few additional points.

Asymptotes

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Definition (Vertical Asymptote)

The line $x = a$ is a vertival asymptote of the graph of f if

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or if

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Definition (Horizontal Asymptote)

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or if

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Vertical Asymptotes of Rational Functions

Theorem

If $f(x) = \dfrac{P(x)}{Q(x)}$ *where* P *and* Q *are polynomial functions, then the line* $x = a$ *is a vertical asymptote of the graph of* f *if* $Q(a) = 0$ *but* $P(a) \neq 0$*.*

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No! It's not! The Q part is 0, but so it the top. There is just a hole in the graph of the function.

Example (Is $x = -1$ a vertical asymptote of $f(x) = \frac{1}{x+1}$?)

Yes! The Q part is 0 while the P part is $1 \neq 0$.

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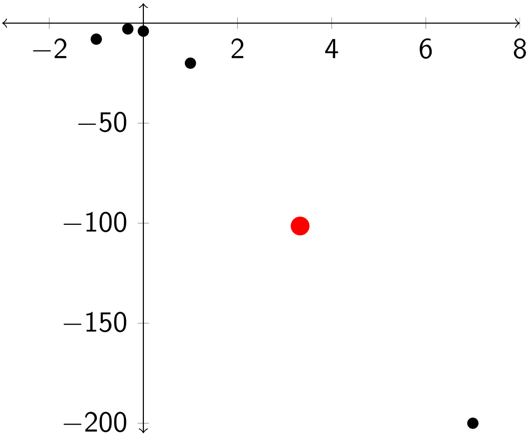
6. The points $(1, -20)$ and $(-1, -8)$ are also on the graph.

Graph



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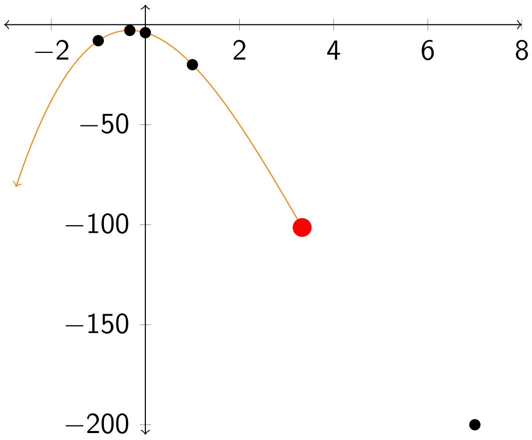

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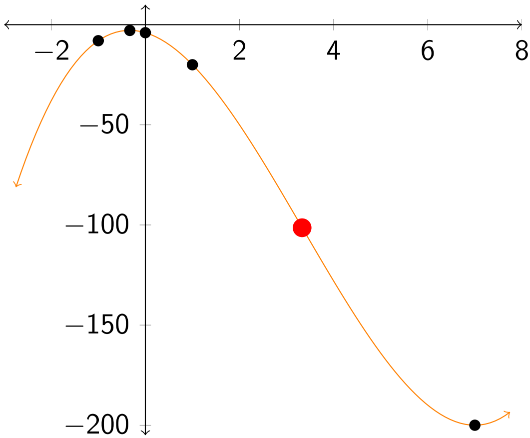

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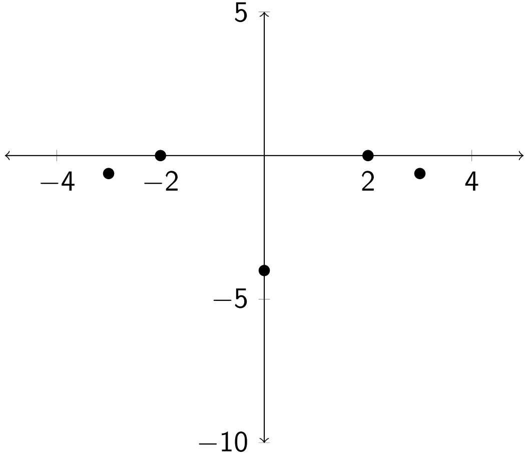

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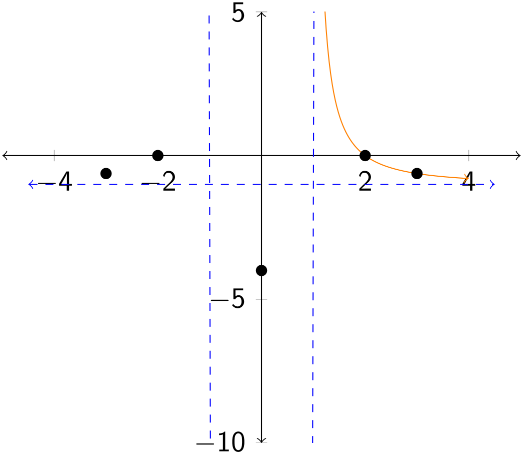

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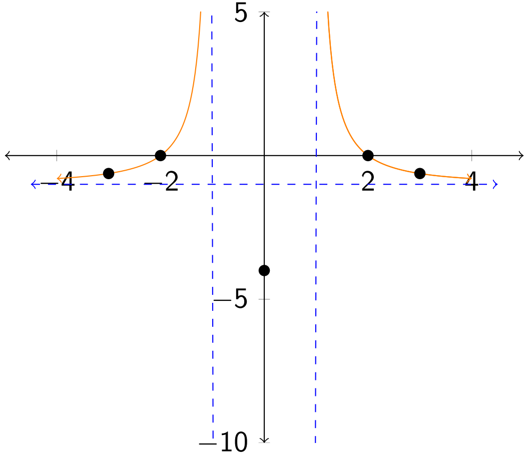

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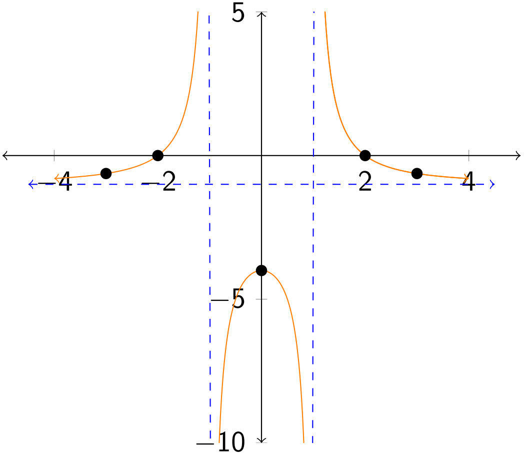

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Theorem (Extreme Value Theorem)

If a function f is continuous on a closed interval $[a, b]$, then f has both an absolute maximum and an absolute minimum value on $[a, b]$.

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Well, an extreme value must either occur at a local extrema or an endpoint, quite obviously.

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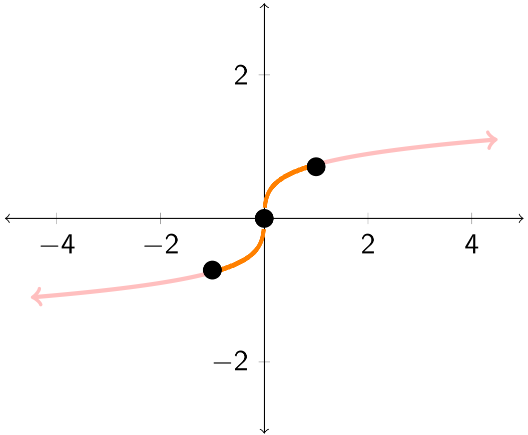
Example

Find the absolute extrema of $f(x) = x^{\frac{1}{5}}$ on $[-1, 1]$.

Well, $f'(x) = \dfrac{1}{5x^{\frac{4}{5}}}$. So, $f'(x) \neq 0$ for any x, but it is undefined at $x = 0$. So $x = 0$ is our only critical number. $f(0) = 0$, $f(1) = 1$ and $f(-1) = -1$. So, our absolute maximum value is $f(1) = 1$ and our absolute minimum value is $f(-1) = -1$.

Graph



Find the absolute extrema of $f(x) = x^3 - 2x^2 - 4x + 4$ on $[0, 3]$.

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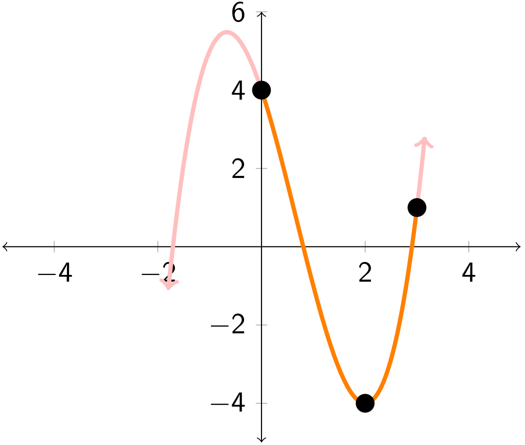
Well, $f'(x) = 3x^2 - 4x - 4 = (3x + 2)(x - 2)$. So, $f'(x) = 0$ for $x = -\frac{2}{3}, 2$. And $f(0) = 4$, $f(2) = -4$ and $f(3) = 1$.

Find the absolute extrema of $f(x) = x^3 - 2x^2 - 4x + 4$ on $[0, 3]$.

Well, $f'(x) = 3x^2 - 4x - 4 = (3x + 2)(x - 2)$. So, $f'(x) = 0$ for $x = -\frac{2}{3}, 2$. And $f(0) = 4$, $f(2) = -4$ and $f(3) = 1$. So, our absolute maximum value is $f(0) = 4$ and our absolute minimum value is $f(2) = -4$.

Graph



Assignment

Read 4.5. Do problems 16, 26, 30, 56, 68 in 4.3 and 8, 21, 37, 48, 82 in 4.4.