QMI Lesson 18:
Antiderivative and Rules of Integration

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What if we wanted to do the opposite, i.e. what if, given a speed fucntion and initial position, how might be find the position function? Naturally, we would want to do the opposite of differentiation! We call this **antidifferentiation**.

Definition (The Antiderivative)

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The function $F(x) = x^3 + 2x + 1$ is an antiderivative of $f(x) = 3x^2 + 2$ because $F'(x) = 3x^2 + 2- = f(x)$.

Antiderivatives Are Not Unique

Show that $F(x) = \frac{1}{3}x^3 - 2x^2 - x - 1$ and $G(x) = \frac{1}{3}x^3 - 2x^2 - x - 9$ are both antiderivatives of $f(x) = x^2 - 4x - 1$.

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Well,

$$F'(x) = x^2 - 4x - 1 = G'(x) = f(x).$$

Moreover, notice that $F_C(x) = \frac{1}{3}x^3 - 2x^2 - x + C$, where C is just some constant is also an antiderivative of $f(x)$. In fact, this is true in general!

Theorem

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Theorem

Let G be an antiderivative of a function f on an interval I. Then every antiderivative F of f on I must be of the form $F(x) = G(x) + C$, where C is some constant. Also, any function F of this form is an antiderivative of f on I.

Prove that $F(x) = x^2 + 1$ is an antiderivative of the function $f(x) = 2x$. Write a general formula for the antiderivative of f.

Antiderivatives of the Function $f(x) = 2x$

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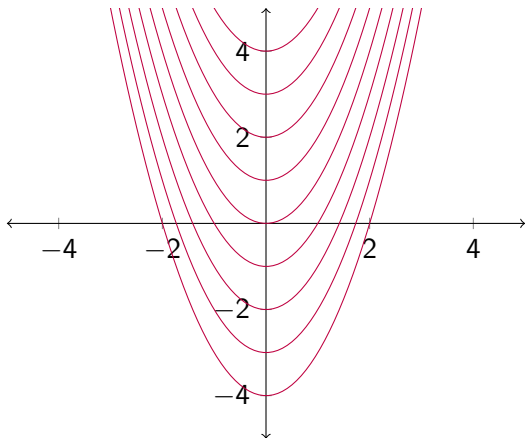Notice, we could simply rewrite this formula as $F(x) = x^2 + C$. Why? Because the constant term C can be made to "swallow up" all other constant terms.

Antiderivatives of the Function $f(x) = 2x$

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The Indefinite Integral

The process of finding all antiderivatives of a function is called antidifferentiation or **integration**. We use the symbol $\int$ to denote the integral operation. Thus, if f is the derivative of F, then

$$\int f(x) \ dx = F(x) + C.$$

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Using this notation, we can write

$$\int 1 + x \ dx = x + \frac{1}{2}x^2 + C,$$

where C is an arbitrary constant.

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Theorem (The Power Rule)

$$\int x^n \ dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1$$

Theorem (Indefinite Integral of a Constant Multiple of a Function)

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Theorem (The Indefinite Integral of $f(x) = x^{-1}$)

$$\int x^{-1} \ dx = \ln |x| + C, \quad C \ a \ constant$$

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For instance, we know that $\int x^2 \ dx = \frac{x^3}{3}$, but if we misapply the rule for the indefinite integral of a constant multiple of a function, we may get

$$\int x^2 \ dx = x^2 \int 1 \ dx = x^2(x) = x^3,$$

which is wrong!

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An initial value problem includes a differential equation (like $f'(x) = 2x - 1$ and an inition condition (like $f(0) = 1$). To solve an initial value problem, simply

1. find a general solution and
2. solve for C using the initial condition.

Let's solve the differential equation from the previous slide where

$$f'(x) = 2x - 1, \quad f(0) = 1.$$

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Then $f(x) = \int 2x - 1 \ dx = x^2 - x + C$. Thus, we have that

$$0^2 - 0 + C = 1 \implies C = 1.$$

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Let's solve the differential equation from the previous slide where

$$f'(x) = 2x - 1, \quad f(0) = 1.$$

Then $f(x) = \int 2x - 1 \ dx = x^2 - x + C$. Thus, we have that

$$0^2 - 0 + C = 1 \implies C = 1.$$

So we have that $f(x) = x^2 - x + 1$ must be our particular solution.

Example

Solve the differential equation given by

$$f'(x) = 3x^2 - 4x + 8, \quad f(1) = 9.$$

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Example

The circulation of *Investor's Digest* is 3000 copies per week currently. The managing editor of this magazine projects a growth rate of $4 + 5t^{\frac{2}{3}}$ copies per week, where t is measured in weeks. This projection is good for the next three years. On the basis of her projection, what will be the circulation of the digest 125 weeks from now?

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so $S(t) = 4t + 3t^{\frac{5}{3}} + 3000$. Therefore,
$S(125) = 4(125) + 3(125^{\frac{5}{3}} + 3000 = 12875.$

Acceleration, Velocity, and Position

A train is accelerating at 2 miles per hour, it's speed at time $t = 0$ is 0 miles per hour, and its initial position at time $t = 1$ is 10 miles down the track. Time is measure in hours. Knowing that the derivative of position gives velocity and that the derivative of velocity gives acceleration, find a formula for the train's position at time t.

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which gives that $s(t) = t^2 + 9$, our particular solution.

Assignment

Read 6.2-6.3. Do problems 4, 8, 20, 36, 38, 50, 54, 58, 62, 70, 100 in 6.1.