QMI Lesson 3: Modeling with Functions

C C Moxley

Samford University Brock School of Business

3 September 2014

Mathematical Models

A mathematical model helps answer problems that arise from real-world situations. Creating and using mathematical models involves four general steps.

Mathematical Models

A mathematical model helps answer problems that arise from real-world situations. Creating and using mathematical models involves four general steps.

1. **Formulate:** From the real-world problem, derive a mathematical statement which captures the way one (dependent) variable relies on another (independent) variable. This is perhaps the hardest step.

Mathematical Models

A mathematical model helps answer problems that arise from real-world situations. Creating and using mathematical models involves four general steps.

1. **Formulate:** From the real-world problem, derive a mathematical statement which captures the way one (dependent) variable relies on another (independent) variable. This is perhaps the hardest step.
2. **Solve:** Use the techniques taught in this course to solve the mathematical problem arising from the real-world problem.

Mathematical Models

A mathematical model helps answer problems that arise from real-world situations. Creating and using mathematical models involves four general steps.

1. **Formulate:** From the real-world problem, derive a mathematical statement which captures the way one (dependent) variable relies on another (independent) variable. This is perhaps the hardest step.
2. **Solve:** Use the techniques taught in this course to solve the mathematical problem arising from the real-world problem.
3. **Interpret:** Translate the solution into its real-world meaning.

Mathematical Models

A mathematical model helps answer problems that arise from real-world situations. Creating and using mathematical models involves four general steps.

1. **Formulate:** From the real-world problem, derive a mathematical statement which captures the way one (dependent) variable relies on another (independent) variable. This is perhaps the hardest step.
2. **Solve:** Use the techniques taught in this course to solve the mathematical problem arising from the real-world problem.
3. **Interpret:** Translate the solution into its real-world meaning.
4. **Test:** Ensure that your real-world solution is an appropriate answer to your original real-world problem.

Mathematical Models

Definition (Polynomial Function)

A polynomial function of degree n is one which can be written in the form

Mathematical Models

Definition (Polynomial Function)

A polynomial function of degree n is one which can be written in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 \quad (a_n \neq 0\ ).$$

Mathematical Models

Definition (Polynomial Function)

A polynomial function of degree n is one which can be written in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 \quad (a_n \neq 0\ ).$$

Here, n must be a non-negative integer.

Mathematical Models

Definition (Polynomial Function)

A polynomial function of degree n is one which can be written in the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 \quad (a_n \neq 0\ ).$$

Here, n must be a non-negative integer. The numbers $a_n, a_{n-1}, \ldots, a_1, a_0$ are called coefficients.

Mathematical Models

Definition (Polynomial Function)

A polynomial function of degree n is one which can be written in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 \quad (a_n \neq 0\ ).$$

Here, n must be a non-negative integer. The numbers $a_n, a_{n-1}, \ldots, a_1, a_0$ are called coefficients.

Definition (Linear Function)

A degree one polynomial is a linear function, i.e. it can be written $f(x) = a_1 x + a_0$ with $a_1 \neq 0$.

Example

Overdraft charges are a major source of revenue for some banks. The following gives the revenue from overdraft fees in billions of dollars from 2004 to 2009.

Year (t)	0	1	2	3	4	5
Revenue (y)	27.5	29	31	34	36	38

Example

Overdraft charges are a major source of revenue for some banks. The following gives the revenue from overdraft fees in billions of dollars from 2004 to 2009.

Year (t)	0	1	2	3	4	5
Revenue (y)	27.5	29	31	34	36	38

Here, t is in years, and $t = 0$ corresponds to year 2004. A linear model giving the approximate projected revenue from overdraft fees is given by

$$f(t) = 2.19t + 27.12 \qquad (0 \leq t \leq 5).$$

Example

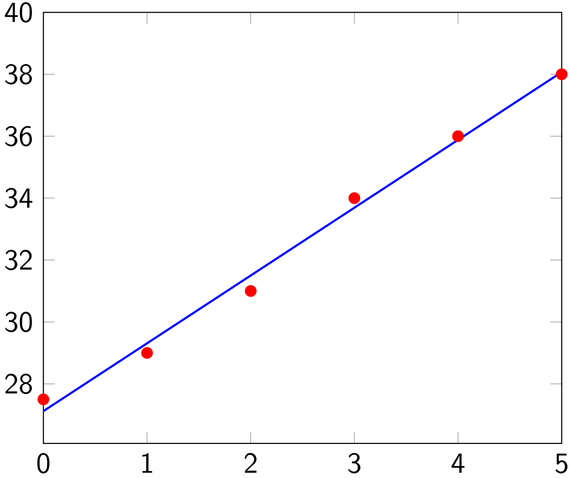
Plot the points from the previous problem. And plot the graph of f over those points.

Example

Plot the points from the previous problem. And plot the graph of f over those points.

Example

Plot the points from the previous problem. And plot the graph of f over those points.



Example, Recall: $f(t) = 2.19t + 27.12$

Assuming that the trend continues, what is the projected revenue in 2012?

Example, Recall: $f(t) = 2.19t + 27.12$

Assuming that the trend continues, what is the projected revenue in 2012?
Well, in 2012, $t = 8$. So $f(8) = 2.19(8) + 27.12 = 44.64$. The projected revenue is \$44.64 billion.

Example, Recall: $f(t) = 2.19t + 27.12$

Assuming that the trend continues, what is the projected revenue in 2012?
Well, in 2012, $t = 8$. So $f(8) = 2.19(8) + 27.12 = 44.64$. The projected revenue is \$44.64 billion.

At what rate did revenue increase from years 2004 to 2009?

Example, Recall: $f(t) = 2.19t + 27.12$

Assuming that the trend continues, what is the projected revenue in 2012?
Well, in 2012, $t = 8$. So $f(8) = 2.19(8) + 27.12 = 44.64$. The projected revenue is $44.64 billion.

At what rate did revenue increase from years 2004 to 2009?
The real rate was $\frac{38-27.5}{5} = 2.1$, but the modeled rate was 2.19 (in billions of dollars).

Quadratic Functions

Definition (Quadratic Function)

A degree two polynomial is a quadratic (parabolic) function, i.e. it can be written in the form $f(x) = a_2x^2 + a_1x + a_0$ with $a_2 \neq 0$.

Quadratic Functions

Definition (Quadratic Function)

A degree two polynomial is a quadratic (parabolic) function, i.e. it can be written in the form $f(x) = a_2x^2 + a_1x + a_0$ with $a_2 \neq 0$.

The graph of a quadratic function is a parabola which opens up if $a_2 > 0$ or down if $a_2 < 0$.

Rational and Power Functions

Definition (Rational Function)

A rational function is the quotient of two polynomial functions.

Rational and Power Functions

Definition (Rational Function)

A rational function is the quotient of two polynomial functions.

Definition (Power Function)

A power function is a function of the form $f(x) = x^r$, where r is any non-zero number.

Rational and Power Functions

Definition (Rational Function)

A rational function is the quotient of two polynomial functions.

Definition (Power Function)

A power function is a function of the form $f(x) = x^r$, where r is any non-zero number.

Is $f(x) = \frac{x^2-3x}{x^{-1}+x}$ a rational function?

Rational and Power Functions

Definition (Rational Function)

A rational function is the quotient of two polynomial functions.

Definition (Power Function)

A power function is a function of the form $f(x) = x^r$, where r is any non-zero number.

Is $f(x) = \frac{x^2 - 3x}{x^{-1} + x}$ a rational function? **Yes!**

Notice: $f(x) = f(x) \cdot \frac{x}{x} = \frac{x^3 - 3x^2}{1 + x^2}$.

Rational and Power Functions

Definition (Rational Function)

A rational function is the quotient of two polynomial functions.

Definition (Power Function)

A power function is a function of the form $f(x) = x^r$, where r is any non-zero number.

Is $f(x) = \frac{x^2 - 3x}{x^{-1} + x}$ a rational function? **Yes!**
Notice: $f(x) = f(x) \cdot \frac{x}{x} = \frac{x^3 - 3x^2}{1 + x^2}$.

Is $f(x) = x^x$ a power function?

Rational and Power Functions

Definition (Rational Function)

A rational function is the quotient of two polynomial functions.

Definition (Power Function)

A power function is a function of the form $f(x) = x^r$, where r is any non-zero number.

Is $f(x) = \frac{x^2-3x}{x^{-1}+x}$ a rational function? **Yes!**
Notice: $f(x) = f(x) \cdot \frac{x}{x} = \frac{x^3-3x^2}{1+x^2}$.

Is $f(x) = x^x$ a power function? **No!**

Supply

Definition (Supply Equation)

This equation expresses the relationship between unit price and the quantity supplied. The graph of this equation is called the supply curve. Often, the independent variable in the supply function is the quantity supplied.

Supply

Definition (Supply Equation)

This equation expresses the relationship between unit price and the quantity supplied. The graph of this equation is called the supply curve. Often, the independent variable in the supply function is the quantity supplied.

It is characteristic of the supply curve to be an increasing function.

Demand

Definition (Demand Equation)

This equation expresses the relationship between unit price and the quantity demanded. The graph of this equation is called the demand curve. Often, the independent variable in the demand function is the quantity demanded.

Demand

Definition (Demand Equation)

This equation expresses the relationship between unit price and the quantity demanded. The graph of this equation is called the demand curve. Often, the independent variable in the demand function is the quantity demanded.

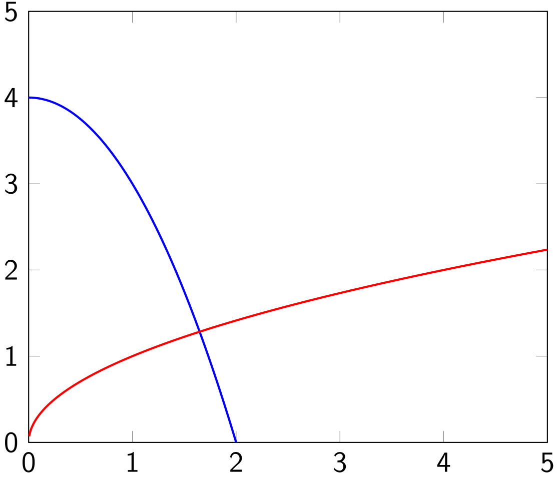
It is characteristic of the demand curve to be a decreasing function.

Equilibrium

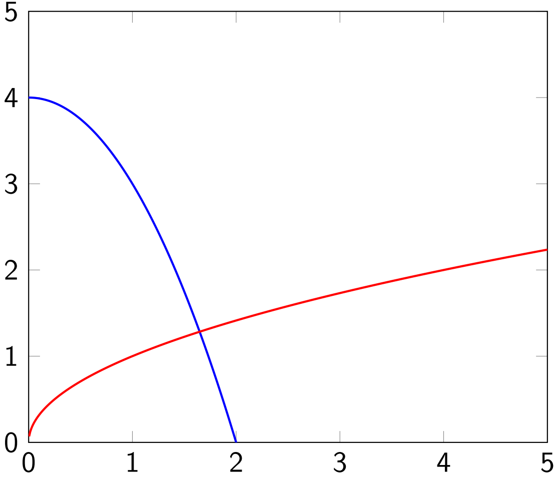
Definition (Market Equilibrium)

Market equilibrium occurs when supply and demand are equal, i.e. it is when the the supply and demand curves intersect. It corresponds to an equilibrium quantity and price.

Supply, Demand, and Equilibrium

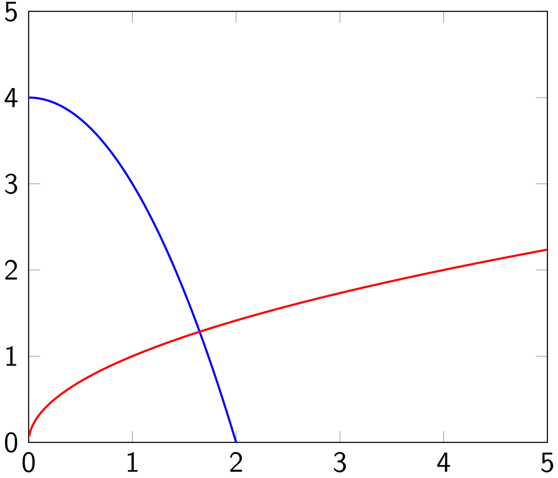


Supply, Demand, and Equilibrium



Which curve is the supply curve?

Supply, Demand, and Equilibrium



Which curve is the supply curve? The demand curve?

Example

The demand function for a certain Bluetooth device is given by $p = d(x) = -0.025x^2 - 05x + 60$, and its supply function is $p = s(x) = 0.02x^2 + 0.6x + 20$, where p is expressed in dollars and x is measured in the thousands of units. Find the equilibrium price and quantity.

Example

The demand function for a certain Bluetooth device is given by $p = d(x) = -0.025x^2 - 05x + 60$, and its supply function is $p = s(x) = 0.02x^2 + 0.6x + 20$, where p is expressed in dollars and x is measured in the thousands of units. Find the equilibrium price and quantity.

We must solve the system of equations

$$p = -0.025x^2 - 05x + 60$$
$$p = 0.02x^2 + 0.6x + 20.$$

Example

Thus, we have

$$-0.025x^2 - 05x + 60 = 0.02x^2 + 0.6x + 20 \implies$$

Example

Thus, we have

$$-0.025x^2 - 05x + 60 = 0.02x^2 + 0.6x + 20 \implies$$
$$0.045x^2 + 1.1x - 40 = 0 \implies$$

Example

Thus, we have

$$-0.025x^2 - 05x + 60 = 0.02x^2 + 0.6x + 20 \implies$$
$$0.045x^2 + 1.1x - 40 = 0 \implies$$
$$45x^2 + 1100x - 40000 = 0 \implies$$

Example

Thus, we have

$$-0.025x^2 - 05x + 60 = 0.02x^2 + 0.6x + 20 \implies$$
$$0.045x^2 + 1.1x - 40 = 0 \implies$$
$$45x^2 + 1100x - 40000 = 0 \implies$$
$$9x^2 + 220 - 8000 = 0 \implies$$

Example

Thus, we have

$$-0.025x^2 - 05x + 60 = 0.02x^2 + 0.6x + 20 \implies$$
$$0.045x^2 + 1.1x - 40 = 0 \implies$$
$$45x^2 + 1100x - 40000 = 0 \implies$$
$$9x^2 + 220 - 8000 = 0 \implies$$
$$(9x + 400)(x - 20) = 0 \implies$$

Example

Thus, we have

$$
\begin{gathered}
-0.025x^2 - 05x + 60 = 0.02x^2 + 0.6x + 20 \implies \\
0.045x^2 + 1.1x - 40 = 0 \implies \\
45x^2 + 1100x - 40000 = 0 \implies \\
9x^2 + 220 - 8000 = 0 \implies \\
(9x + 400)(x - 20) = 0 \implies
\end{gathered}
$$

$x = 20$ is the solution,

Example

Thus, we have

$$
\begin{gathered}
-0.025x^2 - 05x + 60 = 0.02x^2 + 0.6x + 20 \implies \\
0.045x^2 + 1.1x - 40 = 0 \implies \\
45x^2 + 1100x - 40000 = 0 \implies \\
9x^2 + 220 - 8000 = 0 \implies \\
(9x + 400)(x - 20) = 0 \implies \\
x = 20 \text{ is the solution, i.e. the equilibrium quantity.}
\end{gathered}
$$

Example

Thus, we have

$$
\begin{gathered}
-0.025x^2 - 05x + 60 = 0.02x^2 + 0.6x + 20 \implies \\
0.045x^2 + 1.1x - 40 = 0 \implies \\
45x^2 + 1100x - 40000 = 0 \implies \\
9x^2 + 220 - 8000 = 0 \implies \\
(9x + 400)(x - 20) = 0 \implies \\
x = 20 \text{ is the solution, i.e. the equilibrium quantity.}
\end{gathered}
$$

Thus, $p = s(20) = d(20) = -0.025(20)^2 - 0.5(20) + 60 = 40.$

Example

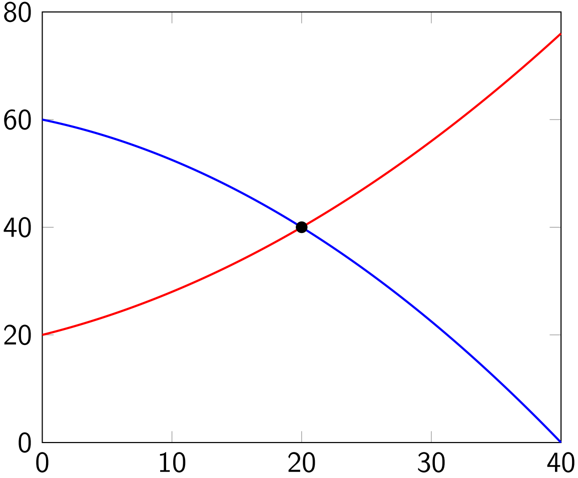
Thus, we have

$$
\begin{gathered}
-0.025x^2 - 05x + 60 = 0.02x^2 + 0.6x + 20 \implies \\
0.045x^2 + 1.1x - 40 = 0 \implies \\
45x^2 + 1100x - 40000 = 0 \implies \\
9x^2 + 220 - 8000 = 0 \implies \\
(9x + 400)(x - 20) = 0 \implies \\
x = 20 \text{ is the solution, i.e. the equilibrium quantity.}
\end{gathered}
$$

Thus, $p = s(20) = d(20) = -0.025(20)^2 - 0.5(20) + 60 = 40.$
Thus, the equilibrium price is \$40 per headset.

Graph of Example

Graph of Example



How to Construct Mathematical Models

These three steps can help you construct a mathematical model.

How to Construct Mathematical Models

These three steps can help you construct a mathematical model.

1. Label each variable in the problem and draw a figure or diagram.

How to Construct Mathematical Models

These three steps can help you construct a mathematical model.

1. Label each variable in the problem and draw a figure or diagram.
2. Find ways of representing one variable in terms of the others. There may be multiple ways of doing this. Pick the most useful one.

How to Construct Mathematical Models

These three steps can help you construct a mathematical model.

1. Label each variable in the problem and draw a figure or diagram.
2. Find ways of representing one variable in terms of the others. There may be multiple ways of doing this. Pick the most useful one.
3. Write a function for the quantity sought. Pay careful attention to the domain of this function.

Example

PlaneX charges \$300 per person if exactly 200 people sign of for a group flight. If more than sign up, the per-person fare is reduced by \$1 for each person. Say x people (where $x \geq 200$) people booked a group flight. White a function for the total revenue realized by PlaneX when x people booked a group flight.

Example

PlaneX charges \$300 per person if exactly 200 people sign of for a group flight. If more than sign up, the per-person fare is reduced by \$1 for each person. Say x people (where $x \geq 200$) people booked a group flight. White a function for the total revenue realized by PlaneX when x people booked a group flight.

Let p be the per-person price.

Example

PlaneX charges \$300 per person if exactly 200 people sign of for a group flight. If more than sign up, the per-person fare is reduced by \$1 for each person. Say x people (where $x \geq 200$) people booked a group flight. White a function for the total revenue realized by PlaneX when x people booked a group flight.

Let p be the per-person price. Then, in terms of x, we have $p = (300 - (x - 200))$

Example

PlaneX charges \$300 per person if exactly 200 people sign of for a group flight. If more than sign up, the per-person fare is reduced by \$1 for each person. Say x people (where $x \geq 200$) people booked a group flight. White a function for the total revenue realized by PlaneX when x people booked a group flight.

Let p be the per-person price. Then, in terms of x, we have $p = (300 - (x - 200)) = 500 - x$.

Example

PlaneX charges \$300 per person if exactly 200 people sign of for a group flight. If more than sign up, the per-person fare is reduced by \$1 for each person. Say x people (where $x \geq 200$) people booked a group flight. White a function for the total revenue realized by PlaneX when x people booked a group flight.

Let p be the per-person price. Then, in terms of x, we have $p = (300 - (x - 200)) = 500 - x$. So, the total revenue $R(x)$ is given by

Example

PlaneX charges \$300 per person if exactly 200 people sign of for a group flight. If more than sign up, the per-person fare is reduced by \$1 for each person. Say x people (where $x \geq 200$) people booked a group flight. White a function for the total revenue realized by PlaneX when x people booked a group flight.

Let p be the per-person price. Then, in terms of x, we have $p = (300 - (x - 200)) = 500 - x$. So, the total revenue $R(x)$ is given by $R(x) = x(500 - x)$.

Example

PlaneX charges \$300 per person if exactly 200 people sign of for a group flight. If more than sign up, the per-person fare is reduced by \$1 for each person. Say x people (where $x \geq 200$) people booked a group flight. White a function for the total revenue realized by PlaneX when x people booked a group flight.

Let p be the per-person price. Then, in terms of x, we have $p = (300 - (x - 200)) = 500 - x$. So, the total revenue $R(x)$ is given by $R(x) = x(500 - x)$.
Domain $=$

Example

PlaneX charges \$300 per person if exactly 200 people sign of for a group flight. If more than sign up, the per-person fare is reduced by \$1 for each person. Say x people (where $x \geq 200$) people booked a group flight. White a function for the total revenue realized by PlaneX when x people booked a group flight.

Let p be the per-person price. Then, in terms of x, we have $p = (300 - (x - 200)) = 500 - x$. So, the total revenue $R(x)$ is given by $R(x) = x(500 - x)$.
Domain $= [200, 500]$.

Examples

Determine if the following are true or false.

Examples

Determine if the following are true or false.

- A polynomial function is a sum of constant multiples of power functions.

Examples

Determine if the following are true or false.

- A polynomial function is a sum of constant multiples of power functions. **False** $f(x) = x^2 + 1$ is a polynomial, but

Examples

Determine if the following are true or false.

- A polynomial function is a sum of constant multiples of power functions. **False** $f(x) = x^2 + 1$ is a polynomial, but 1 is not a constant multiple of a power function!
- If $r > 0$, then the power function $f(x) = x^r$ is defined for all values of x.

Examples

Determine if the following are true or false.

- A polynomial function is a sum of constant multiples of power functions. **False** $f(x) = x^2 + 1$ is a polynomial, but 1 is not a constant multiple of a power function!
- If $r > 0$, then the power function $f(x) = x^r$ is defined for all values of x. **No.** Take, for instance $r = 1/2$.

Assignment

Read 2.4. Do problems 6, 12, 18, 26, 38, 48, 62, 66, 74, 80, 82 in 2.3.