QMI Lesson 7: Some Differentiation Rules

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First, we'll go over each rule and its proof. Then we'll move on to examples.

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$\frac{1}{g(x+h)(g(x)} \cdot [g(x)\frac{f(x+h)-f(x)}{h} - f(x)\frac{g(x+h)-g(x)}{h}]$. And passing through the limit on both sides of the equality proves the theorem.

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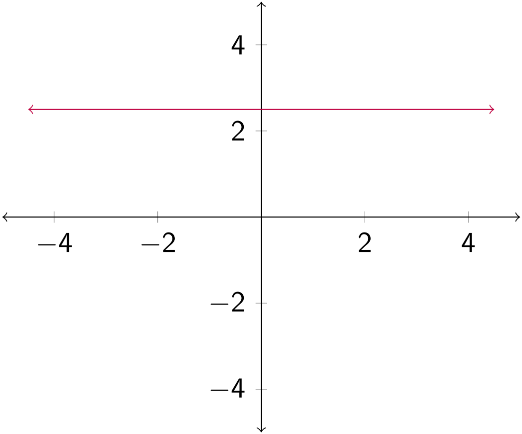

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What is the rate of change in the population after 2 months? 6 months? What is the final population after implementation of the breeding program?

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For instance, if we used this incorrect rule, we would calculate for $h(x) = \frac{x}{x^2+1}$ that

$$h'(x) = \frac{1}{2x},$$

but we know this is wrong since we already calculated

$$h(x) = \frac{1-x^2}{(x^2+1)^2}.$$

Assignment

Read 3.3-3.4. Do problems 28, 31, 36, 38, 42, 50, 70, 78 in 3.1 and 12, 26, 42, 62, 68 in 3.2.