Practice Test 1
BUSA130
25 Sept 2014

This exam is graded out of 100 points. Show all work necessary to solve the problems. You have 65 minutes.

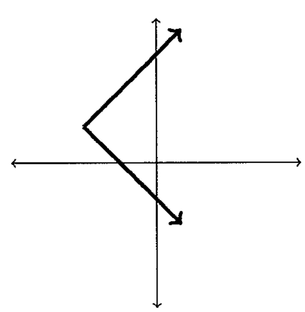
1) Determine if the limit exists. If it does, give its value. (4pts each)

- $\lim\limits_{x\to\infty} \frac{x^2-2x+1}{-x^2+1} =$ -1 because the degree of the numerator = the degree of the denominator and because the ratio of the leading coefficients is -1.
- $\lim\limits_{x\to1} \frac{1}{x^2-1} =$ Does not exist — there is a vertical asymptote at $x=1$.

  $\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)}$ ← nothing cancels, so there is a vertical asymptote.
- $\lim\limits_{h\to0} \frac{\sqrt{4-h}-2}{3h} = \ldots$ Well, $\frac{\sqrt{4-h}-2}{3h} \cdot \frac{\sqrt{4-h}+2}{\sqrt{4-h}+2} = \frac{4-h-4}{3h(\sqrt{4-h}+2)} = \frac{-\cancel{h}}{3\cancel{h}(\sqrt{4-h}+2)}$

So, $\lim\limits_{h\to0} \frac{\sqrt{4-h}-2}{3h} = \lim\limits_{h\to0} \frac{-1}{3(\sqrt{4-h}+2)} = -\frac{1}{12}$

2) Does this graph define a function from x to y? Why or why not? Does it define a function from y to x? Why or why not? (6pts)



Because it fails the vertical line test, the graph does not define a function from x to y. But because it passes the horizontal line test, it does define a function from y to x.

Because of this, both functions will be continuous on their entire domains, i.e. everywhere they are defined.

3) Find the values for which the function is defined and continuous. You may assume that the square root function is continuous and that the composition of two continuous functions is also continuous. (5pts each)

- $f(x) = \dfrac{x^2+\sqrt{x}}{3x^2-18x-21}$ Well, $\sqrt{x} \Rightarrow x \geq 0$. We also cannot divide by 0,

So $3x^2-18x-21 \neq 0 \Rightarrow 3(x^2-6x-7)=x \Rightarrow (x-7)(x+1) \neq 0$
$\Rightarrow x \neq -1, 7$. So the function is defined & continuous on $[0,7)\cup(7,\infty)$.

- $h(t) = \sqrt{\dfrac{x^2}{x^2-1}}$ We must have $x^2-1 \neq 0 \Rightarrow (x+1)(x-1)=0 \Rightarrow x \neq 1,-1$.

Also, $\dfrac{x^2}{x^2-1} \geq 0 \Rightarrow$

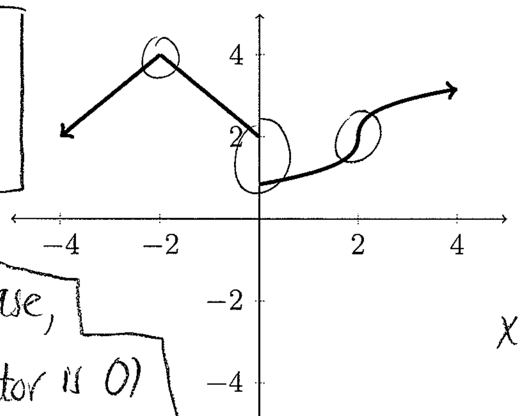
$+ \quad - \quad - \quad +$

$-1 \quad 0 \quad 1$

$\Rightarrow$ the function is contin. and defined on $(-\infty,-1)\cup(1,\infty)$.

to calculate where a rational expression is non-negative you must find all the roots of its numerator (in this case, the only root of the numerator is 0) and denominator (in this case, the only roots of the denominator are 1, -1) and then break the number line over these points to see where the function will be non negative.

4) For what values of x is the function whose graph is given below not differentiable? Explain why the function is not differentiable at these points. (6 pts)



Not differentiable at:

$x=-2$ (sharp turn, corner)

$x=0$ (discontinuity)

$x=2$ (vertical tangent line).

5) The percentage of families that were married couples without children between 1970 and 2000 is given by

$$P(t) = \frac{55.1}{t^{0.29}}, \qquad (1 \leq t \leq 4)$$

where t is in decades and t=1 corresponding to 1970. What was the percentage of married families without children in 1980? In 2000? What was the rate of change of the percentage of married families without children in 1980? (Give units.) (8pts)

$$P(2) = \frac{55.1}{(2)^{0.29}} \approx 45.07\,\%$$

$$P(4) = \frac{55.1}{4^{(0.29)}} \approx 36.86\,\%$$

$$P'(t) = \frac{d}{dt}\left(\frac{55.1}{t^{0.29}}\right) = 55.1\,\frac{d}{dt}(t^{-0.29}) = 55.1(-0.29)t^{-1.29}$$

So, $P'(2) \approx 55.1(-0.29)(2)^{-1.29} \approx \boxed{-6.53 \text{ percent per decade}}$

this part at bottom

6) Find $f'(x)$ for the following. (5pts each) Also, for the **first function only**, find the equation of the tangent line at the point $(0,0)$. (4pts)

- $f(x) = 3x\sqrt{x^2+1}$ $\Longrightarrow f'(x) = (3x)' \cdot \sqrt{x^2+1} + 3x(\sqrt{x^2+1})'$

you do not have to simplify your answers.

$$= \boxed{3\sqrt{x^2+1} + 3x(\tfrac{1}{2})(2x)(x^2+1)^{-\frac{1}{2}}}$$

- $f(x) = \dfrac{-x^2+x^3}{2x-2}$ $\Longrightarrow f'(x) = \dfrac{(2x-2)(-x^2+x^3)' - (-x^2+x^3)(2x-2)'}{(2x-2)^2}$

$$= \frac{(2x-2)(-2x+3x^2) - (-x^2+x^3)(2)}{(2x-2)^2}$$

- $f(x) = (3x^2-2x+1)^3$ $\Longrightarrow f'(x) = 3(3x^2-2x+1)^2(6x-2)$

- $f(x) = \left[\dfrac{2x+1}{-x+3}\right]^2$ $\Longrightarrow f'(x) = 2\left(\dfrac{2x+1}{-x+3}\right)\left(\dfrac{2x+1}{-x+3}\right)'$

$$= 2\left(\frac{2x+1}{-x+3}\right)\left(\frac{(-x+3)2 - (2x+1)(-1)}{(-x+3)^2}\right)$$

$$f'(0) = 3\sqrt{0^2+1} + 3(0)(\tfrac{1}{2})(2(0))(0^2+1)^{-\frac{1}{2}} = 3$$

So $y - 0 = 3(x-0) \Longrightarrow y = 3x$ is the equation of the tangent line at $(0,0)$.

$$p = \sqrt{10-0.025x} \Longrightarrow p^2 = 10 - 0.025x \Longrightarrow$$

because of the domain given before.

$$\frac{10-p^2}{0.025} = x \Longrightarrow \boxed{400 - 40p^2 = x = f(p)}, \quad 0 \le p \le 10$$

7) The demand function for a children's tricycle is given by

we need to make this a function of price!

$$p = \sqrt{10 - 0.025x}, \quad (0 \le x \le 400),$$

where p is the unit price in hundreds of dollars and x is the quantity demanded per month. Compute the elasticity of demand and determine the range of prices corresponding to inelastic, unitary, and elastic demand. (10pts)

$$E(p) = \frac{-p\, f'(p)}{f(p)}, \text{ so } E(p) = \frac{-p(-80p)}{400-40p^2}$$

$$\Longrightarrow E(p) = \frac{80p^2}{400-40p^2} = \frac{2p^2}{10-p^2}$$

Now, to compute ranges of elastic, inelastic, and unitary demand, we must solve: $E(p) = 1 = \dfrac{2p^2}{10-p^2} \Longrightarrow$

$$0 = \frac{2p^2}{10-p^2} - 1 = \frac{2p^2-(10-p^2)}{10-p^2} = \frac{3p^2-10}{10-p^2} \Longrightarrow p = \sqrt{\frac{10}{3}}$$

So, unitary is at $p = \sqrt{\frac{10}{3}}$, elastic on p in $\left(\sqrt{\frac{10}{3}}, \sqrt{10}\right) \cup (\sqrt{10}, 10]$ and inelastic on p in $\left[0, \sqrt{\frac{10}{3}}\right)$.

8) Assume $f(x) \cdot g(y) = 2x$, $f(x) \neq 0$, and $g'(y) \neq 0$. Show that $\dfrac{dy}{dx} = \dfrac{2 - f'(x)g(y)}{f(x)g'(y)}$.
(Hint: differentiate the first equation with respect to x using implicit differentiation.) (12pts)

If $f(x) \cdot g(y) = 2x$, then $f'(x) \cdot g(y) + f(x)\, g'(y) \dfrac{dy}{dx} = 2$

$$\Longrightarrow 2 - f'(x)g(y) = f(x)g'(y)\frac{dy}{dx}$$

$$\Longrightarrow \frac{2-f'(x)g(y)}{f(x)\,g'(y)} = \frac{dy}{dx}$$

✓

9) Use differentials to approximate the value of $\frac{1}{\sqrt[3]{8.04}} + \sqrt[3]{8.04}$. Hint: Let

$$f(x) - \frac{1}{\sqrt[3]{x}} + \sqrt[3]{x}$$

and calculate dy with $x = 8$ and $dx = 0.04$. Also, remember that $\Delta y \approx dy$. **Do not round!** (12pts)

$$\Delta y \approx dy\Big|_{x=8,\, dx=0.04} = \left(\frac{-1}{3(x^{\frac{4}{3}})} + \frac{1}{3x^{2/3}}\right)\Delta x\Bigg|_{x=8,\, dx=0.04}$$

$$= \left(\frac{-1}{3(16)} + \frac{1}{3(4)}\right)0.04 = 0.0025.$$

So $$\frac{1}{\sqrt[3]{8.04}} + \sqrt[3]{8.04} \approx \frac{1}{\sqrt[3]{8}} + \sqrt[3]{8} + 0.0025 = 2.5025$$

A bonus question will be drawn from the syllabus! (3pts)