Practice Test 3
BUSA130
25 Nov 2014

This exam is graded out of 100 points. Show all work necessary to solve the problems. You have 65 minutes.

1) Find the **second** derivative of $f(x) = 4xe^{-x}$. (15pts)

$$f'(x) = \frac{d}{dx}(4x) \cdot e^{-x} + \frac{d}{dx}(e^{-x}) \cdot 4x = 4e^{-x} + (-e^{-x})(4x)$$

$$= 4(e^{-x})(1-x)$$

$$f''(x) = 4\left[\frac{d}{dx}(e^{-x}) \cdot (1-x) + \frac{d}{dx}(1-x) \cdot (e^{-x})\right]$$

$$= 4\left(-e^{-x}(1-x) - e^{-x}\right) = 4(e^{-x})(x-2)$$

2) Find the absolute extrema of $f(x) = 2x - \ln(x^2)$ on $[\frac{1}{2},3]$. (20pts)

$$f'(x) = 2 - \frac{2x}{x^2}$$

$$0 = 2 - \frac{2x}{x^2} \implies 2 = \frac{2}{x} \implies x = 1 \quad \text{critical pt.}$$

$$f''(x) = \frac{d}{dx}\left[2 - \frac{2}{x}\right] = \frac{2}{x^2}$$ ← this is positive at $x=1$ and on the whole real line, so

$(1,2)$ ← is an absolute minimum.

<u>NOTE</u>: You can also use the <u>extreme</u> <u>value</u> <u>theorem</u>!!

3) Show $\int f(x) \cdot g(x)\ dx \neq \left(\int f(x)\ dx\right) \cdot \left(\int g(x)\ dx\right)$. (Hint: Use $g(x) = f(x) = 1$.) (15pts)

$$\int 1 \cdot 1\, dx = \int 1\, dx = x + c$$

whereas

$$\underbrace{\int 1\, dx}_{= x} \cdot \underbrace{\int 1\, dx}_{= x} = x^2 + c$$

these two expressions are not identically equal.

4) Find the indefinite integral $\int \left(\frac{-2}{x+2} + x\right)\ dx$. (15pts)

$$\int \left(\frac{-2}{x+2} + x\right) dx = \int \frac{-2}{x+2}\, dx + \int x\, dx$$

$$= -2 \int \frac{1}{x+2}\, dx + \int x\, dx \underset{\substack{\uparrow \\ \text{let } u = x+2 \\ du = dx}}{=} -2 \int \frac{1}{u}\, du + \int x\, dx$$

$$= -2 \ln|u| + \frac{x^2}{2} + c \underset{\substack{\uparrow \\ \text{sub. back} \\ \text{in}}}{=} -2 \ln|x+2| + \frac{x^2}{2} + c$$

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5) Estimate the area under the curve $f(x) = \frac{1}{x+1}$ on the interval [0,2] using 4 subintervals of uniform length and the right endpoints of these intervals. (15pts)

$$A \approx (f(x_1) + f(x_2) + f(x_3) + f(x_4)) \underbrace{(0.5)}_{\Delta x} =$$

$$(f(0.5) + f(1) + f(1.5) + f(2))(0.5) = (1.9)(0.5) = 0.95$$

6) Sunbeam determined that the daily marginal revenue of selling their microwaves is $R'(x) = -0.12x + 50$, where x is the number of units and R' is measured in dollars per unit. Using the Fundamental Theorem of Calculus, find the total daily revenue generated from selling 300 microwaves. (20pts)

revenue from selling first 300. $= \int_0^{300} -0.12x + 50 \, dx = -0.06x^2 + 50x \Big|_0^{300} = 9600 - 0 = \$9600$

There will be a bonus question! ($\geq$3pts)