

Problem 18, Section 2.5

Find the slope of the tangent line to the graph of the function $f(x) = \frac{3}{2x}$ at the point $(1, \frac{3}{2})$. Determine the equation of the tangent line at that point.

There are **two** ways of doing this problem.

The first is calculating (using the four-step method) $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ and plugging in $x = 1$ into the expression you get for $f'(x)$.

We will do this problem the second way, i.e. by directly calculating $f'(1)$.

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{\frac{3}{2(1+h)} - \frac{3}{2}}{h} = \lim_{h \rightarrow 0} \frac{\frac{3 - (3+3h)}{2(1+h)}}{h} = \lim_{h \rightarrow 0} \frac{-3h}{2(1+h)(h)} = \lim_{h \rightarrow 0} \frac{-3}{2(1+h)} = -\frac{3}{2}.$$

Thus, the slope of the tangent line at $(1, \frac{3}{2})$ is $-\frac{3}{2}$. We may now write the equation of the tangent line using slope-intercept form.

$$y - \frac{3}{2} = -\frac{3}{2}(x - 1) \implies y = -\frac{3}{2}x + 3.$$

