Problem 24, Section 3.7

Use differentials to approximate $\sqrt[4]{81.6}$.

We are trying to approximate the function $f(x) = \sqrt[4]{x}$ for $x = 81.6$. We want to do this using the y differential, i.e. dy. Because

$$dy = f'(x)dx,$$

we have that $f(81.6) \approx f(81) + dy\Big|_{x=81,dy=0.6} = f(81) + f'(x)dx\Big|_{x=81,dy=0.6} = f(81) + f'(81)(0.6)$. And we have

$$f'(x) = \frac{1}{4}x^{-\frac{3}{4}} = \frac{1}{4x^{\frac{3}{4}}}.$$

Therefore,

$$f(81.6) \approx f(81) + \frac{1}{4(27)} \cdot (0.6) = \sqrt[4]{81} + \frac{1}{4(27)} \cdot (0.6) = 3.00\overline{555}.$$