

Problem 26, Section 4.3

Find the horizontal and vertical asymptotes of  $h(x) = \frac{2 - x^2}{x^2 + x}$

Horizontal asymptotes occur at the limits at infinity. To find them, we must calculate

$$\lim_{x \rightarrow \pm\infty} h(x) = -1,$$

since the degrees of the numerator and denominator match and since the ratio of the leading coefficients is  $-1$ . So we have  $y = -1$  is the horizontal asymptote. And to calculate the vertical asymptote, we need to find where the numerator is non-zero and the denominator is zero, i.e. we must solve

$$x^2 + x = 0.$$

This is equivalent to solving  $x(x+1) = 0$  which means that  $x = 0$  and  $x = -1$  are our vertical asymptotes because the numerator is non-zero at  $x = 0$  and at  $x = -1$ .