Problem 36, Section 2.1

Find the domain of $f(x) = \dfrac{\sqrt{x-1}}{(x+2)(x-3)}$.

Well, because we cannot divide by zero, we must have that

$$(x+2)(x-3) \neq 0$$

and this implies that

$$(x+2) \neq 0, \qquad (x-3) \neq 0.$$

Solving, we get that $x \neq -2, 3$. But, we also cannot take the square root of a negative number, meaning that the *inside* of the square root must be non-negative. We write this as

$$x - 1 \geq 0.$$

This means that in addition to the above, we must have that $x \geq 1$. We, then, need for x to be greater than or equal to 1 and not equal to -2 or 3. (But notice that x being greather than or equal to 1 means that it is *automatically* not equal to -2 since that is less than 1.) So, we can reduce these conditions to x being greather than or equal to 1 and not equal to 3. We write this in interval notiation as

$$[1, 3) \cup (3, \infty).$$