Problem 4, Section 4.5

A farmer has 3000 yards of fencing to enclose an area along a river with two separate enclosures. (See the figure in Problem 4 on page 319 of the text.) Maximize the area of the two enclosures and give the dimensions of the enclosures.

We need to maximize $A = xy$ subject to $3000 = 3x + y$, so we need to maximize $A = x(3000 - 3x) = 3000x - 3x^2$. Therefore, $A' = 0$ when $0 = 3000 - 6x$, meaning we have a critical number at $x = 500$. We are maximizing over the interval $[0, 1000]$, so testing, we get $A(0) = 0$, $A(1000) = 0$, and $A(500) = 750000$, so $x = 500$, $y = 1500$ maximizes our area at 750000 square yards.