Problem 52, Section 3.3

Given $y = 2u^2 + 1$ and $u = x^2 + 1$, find $\frac{dy}{du}$, $\frac{du}{dx}$, and $\frac{dy}{dx}$.

We use the chain rule! Obviously y can be written as a function of x even though it is currently written as a function of u. We have $y(u) = y(u(x))$. So it's a composite function, and by the chain rule, we have $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

Taking the derivatives, we get

$$\frac{dy}{du} = 4u$$

and

$$\frac{du}{dx} = 2x.$$

Thus,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 4u(2x).$$

But because $u = x^2 + 1$, we get

$$\frac{dy}{dx} = 4(x^2 + 1)(2x) = 8x(x^2 + 1) = 8x^3 + 8x.$$