Problem 76, Section 2.4

Calculate $\lim\limits_{x\to\infty} \dfrac{2x^2 + 3x + 1}{x^4 - x^2}$.

There are two ways of doing this problem. You could notice that the degree of the polynomial in the denominator is greater than the degree of the polynomial in the numerator, and apply the theorem in the lecture notes which says that the limit of such a rational function as $x \to \pm\infty$ is **always 0**! Or, you could do the following calculation.

$$\lim\limits_{x\to\infty} \frac{2x^2 + 3x + 1}{x^4 - x^2} = \lim\limits_{x\to\infty} \frac{2x^2 + 3x + 1}{x^4 - x^2} \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}} = \lim\limits_{x\to\infty} \frac{\frac{2}{x^2} + \frac{3}{x^3} + \frac{1}{x^4}}{1 - \frac{1}{x^2}} = \frac{0 + 0 + 0}{1 - 0} = 0.$$

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