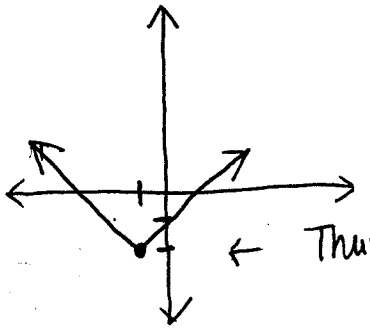
Practice Test 2
BUSA130
7 Nov 2014

This exam is graded out of 100 points. Show all work necessary to solve the problems. You have 65 minutes.

1) Find any relative extrema of the following functions. Remember a relative extrema is a **point** and should be written as $(x, f(x))$. (10pts each)

- $f(x) = |x+1| - 2$. (Hint: Look at the graph!) ← This is the absolute value function shifted left one & down two.



← Thus there is a relative min at $(-1,-2)$.

- $g(x) = x^3 - 3x^2 - 9x + 3$.

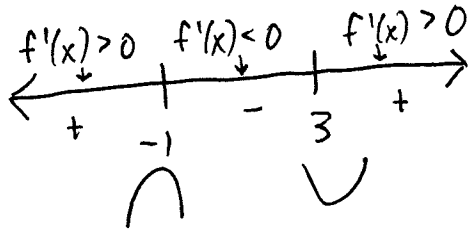
$g'(x) = 3x^2 - 6x - 9$

$g'(x) = 0$ when $3x^2 - 6x - 9 = 0$, i.e. when

$x^2 - 2x - 3 = 0 \Rightarrow$

$(x-3)(x+1) = 0 \Rightarrow x = -1, 3$

test the sign of $f'(x)$ on the intervals →

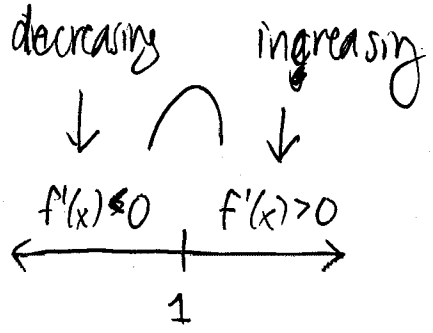


$\therefore$ a local/relative max occurs at $(-1,8)$
a local/relative min occurs at $(3,-24)$

2) Sketch the graph of $f(x) = x - 2\sqrt{x}$. (20pts)

- Domain : $[0, \infty)$
- no vert. or horizontal asymptotes.
- $f(0) = 0$ ← the x- and y- intercept.

$f'(x) = 1 - 2(\frac{1}{2})x^{-\frac{1}{2}} = 1 - \frac{1}{\sqrt{x}}$



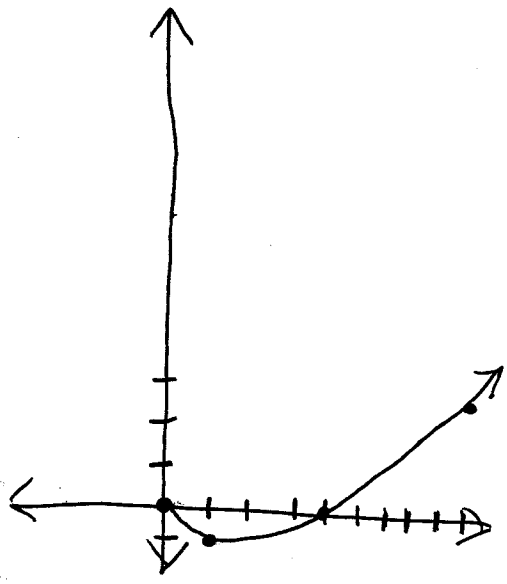
$f'(x) = 0$ when $1 = \frac{1}{\sqrt{x}} \Rightarrow x = 1$

$\Longrightarrow$ $(1, -1)$ is a local minimum

- $f''(x) = (1 - \frac{1}{\sqrt{x}})' = 0 - (x^{-\frac{1}{2}})' = \frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2\sqrt{x^3}}$

$f''(x) > 0$ on the domain of f, so f is concave up everywhere

- extra points : $(4, 0)$ ← another x-intercept.
  $(9, 3)$



3) A ferry fueled by diesel gets $\frac{10}{x}$ nautical miles per gallon of fuel if it is travelling at x nautical miles per hour (where x is between 5 and 10 nautical miles per hour). If the price of diesel is \$5 per gallon and the ferry's crew has a total wage of \$64 per hour, at what speed should the ferry travel to minimize costs? What is this minimal cost rate? (15pts)

$C(x) = \frac{\text{Cost}}{\text{mile}} = \frac{5x}{10} + \frac{64}{x}$

cost of fuel per mile; cost of labor per mile

$C'(x) = \frac{1}{2} - \frac{64}{x^2} = 0$ when $\frac{1}{2} = \frac{64}{x^2}$

$\Rightarrow x^2 = 128 \Rightarrow x \approx 11.3137$ mph. but this is not in our domain $[5, 10]$.

So we test $C(5) = 15.3$, $C(10) = 11.4$ you should have the ferry travel at 10 mph.

4) Determine if it is better to receive a 6% annual interest rate compounded daily or a 6.15% annual interest rate with no compounding. (10pts)

$$r_{eff} = \left(1 + \frac{0.06}{365}\right)^{365} - 1 \cong 0.06183$$

6% compounded daily is better.

$$r_{eff} = (1 + 0.0615) - 1 = 0.0615$$

5) Use the laws of logarithms to expand and simplify the expression $\log \frac{2(10^x)}{x^2}$. (10pts)

$$= \log 2 + x \log 10 - 2 \log x = \log 2 + x - 2 \log x$$

6) Solve $4^x = \frac{1}{16^{-x^2}}$ for x. (10pts)

$\Rightarrow 4^x = 16^{x^2} \Rightarrow 4^x = (4)^{2x^2} \Rightarrow$

$2x^2 = x \Rightarrow 2x^2 - x = 0 \Rightarrow$

~~$x(2-x) = 0 \Rightarrow$~~ ~~$x = 0, 2$~~

$x(2x-1) = 0 \Rightarrow x = 0, \frac{1}{2}$

7) The differential dy is given by $dy = f'(x)dx$. If $f(x) = \sqrt[4]{x}$, use the differential to estimate the value of $\sqrt[4]{16.5}$ by calculating $f(16) + \left(dy \Big|_{x=16, dx=0.5} \right)$. (15pts)

$$dy = \frac{1}{4} x^{-\frac{3}{4}} dx$$

$$f(16.5) \approx f(16) + \frac{1}{4}(16)^{-\frac{3}{4}}(0.5)$$

$$= 2 + \frac{1}{4}\left(\frac{1}{8}\right)\left(\frac{1}{2}\right) = 2.015625$$

There will be a bonus question! (3pts)