QMI Lesson 13: Optimization

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4. Optimize the function f over its domain using the methods of Section 4.4.

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2. We wish to maximize the area $A = lw$ of the enclosure.
3. How do we relate the length to the width? We must do so because we need an equation in a **single** variable.

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The Metro Transit Authority (MTA) operates a subway line for commuters from a certain suburb to downtown. An average of 6000 people take the train per day, paying \$3.00 per ride. The MTA is considering raising the fare to \$3.50 per ride, but knows that for every \$0.50 increase in fare, an average of 1000 people will choose not to take the train. Show that this increase will reduce revenue, and find the maximum fare increase that will not result in a loss of revenue.

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Example

The Metro Transit Authority (MTA) operates a subway line for commuters from a certain suburb to downtown. An average of 6000 people take the train per day, paying \$3.00 per ride. The MTA is considering raising the fare to \$3.50 per ride, but knows that for every \$0.50 increase in fare, an average of 1000 people will choose not to take the train. Show that this increase will reduce revenue, and find the maximum fare increase that will not result in a loss of revenue.

1. Well, we are concerned with the number of people riding the train x, the fare p, and the revenue R.
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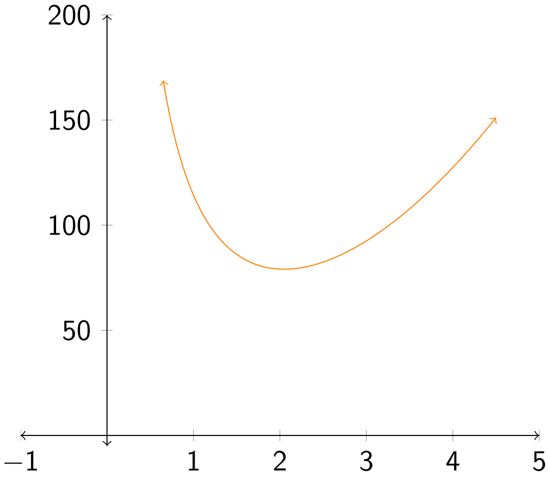
Graph



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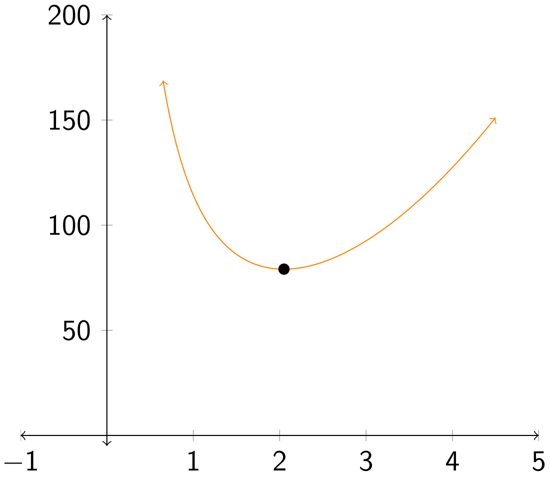

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A motorcycle retailer sells Shadow 250cc motorcycles exclusively. They estimate that the demand for these motorcycles is 10,000 per year and that they are sold at a uniform rate throughout the year. The cost of ordering a shipment is \$10000 and the yearly cost of storing one motorcycle is \$200. How large should a shipment be and how often should orders be made to minimize ordering plus storage costs?

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2. We want to minimize total cost $C = C_{shipping} + C_{storage}$.

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3. We may assume that the orders are placed in such a way that an order arrives exactly as the last motorcycle is being sold. This means that $C_{storage} = 200(\frac{x}{2}) = 100x$. Now, since there need to be 10,000 motorcycles to meet the demand, we need $\frac{10000}{x}$ shippments, meaning $C_{shipping} = 10000(\frac{10000}{x}) = \frac{10^8}{x}$.

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A motorcycle retailer sells Shadow 250cc motorcycles exclusively. They estimate that the demand for these motorcycles is 10,000 per year and that they are sold at a uniform rate throughout the year. The cost of ordering a shipment is \$10000 and the yearly cost of storing one motorcycle is \$200. How large should a shipment be and how often should orders be made to minimize ordering plus storage costs?

3. We may assume that the orders are placed in such a way that an order arrives exactly as the last motorcycle is being sold. This means that $C_{storage} = 200(\frac{x}{2}) = 100x$. Now, since there need to be 10,000 motorcycles to meet the demand, we need $\frac{10000}{x}$ shippments, meaning $C_{shipping} = 10000(\frac{10000}{x}) = \frac{10^8}{x}$. So,

$$C(x) = 100x + \frac{10^8}{x}.$$

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4. And $C''(x) = \frac{2(10^8)}{x^2}$ which is positive for all x in our domain $[1, \infty)$. Thus, $x = 1000$ corresponds to an absolute minimum. So, orders should size 1000 and should be placed 10 times a year (or every 36.5 days).

Assignment

Read 5.1-5.2. Do problems 4, 8, 16, 26, 28, 30, 32, 34 in 4.5.