QMI Lesson 14:
Exponential and Logarithmic Functions

C C Moxley

Samford University Brock School of Business

Exponential Function: A Motivating Example

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What would happen if, every six months, you were paid half of your yearly interest and were allowed to reinvest it? How much would you have after a year? This is called **compounding** your interest twice per year.

$$\underbrace{1000\left(\frac{0.06}{2}\right)}_{\text{interest1}} + \underbrace{\left(1000\left(\frac{0.06}{2}\right) + 1000\right)\left(\frac{0.06}{2}\right)}_{\text{interest2}} + \underbrace{1000}_{\text{principal}} = 1060.9.$$

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where $e \approx 2.718$ is the natural constant. This is **continuous compounding**!

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Exponential functions are useful, as we've seen, in calculating values of investments, and they are also used in modeling population change, bacteria colonization, and many other real-world applications.

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Domain $= (-\infty, \infty)$. And $f(1) = 2$, $f(\frac{1}{2}) = \sqrt{2}$, $f(0) = 1$, and $f(-\frac{2}{3}) = \frac{1}{\sqrt[3]{4}}$.

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5. $\left(\dfrac{a}{b}\right) = \dfrac{a^x}{b^x}$.

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Use the rules above to calculate the following (without a calculator).

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If $b^x = b^z$, then $x = z$. The converse is also true. Here, $b > 0, b \neq 1$.

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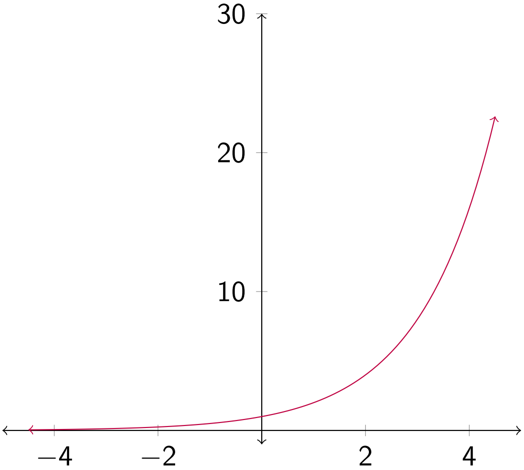
See the graph on the next slide.

Graph of $f(x) = 2^x$



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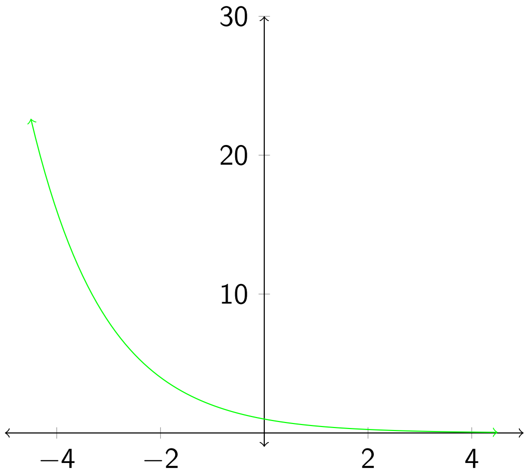
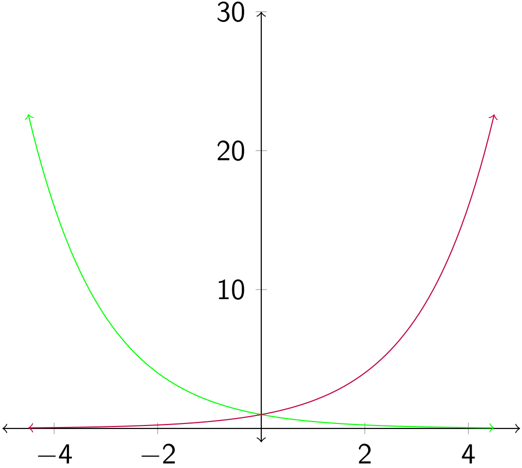
See the graph on the next slide.

Graph of $f(x) = (\frac{1}{2})^x$



Graphs of $f(x) = (\frac{1}{2})^x$ and $g(x) = 2^x$



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Definition (The Value of e^x)

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$$b^y = x.$$

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$$\log_b x = y$$

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Lemma (Laws of Logarithms)

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Note: $\frac{\log_b m}{\log_b n} \neq \log_b m - \log_b n.$

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Examples

Given that $\log 2 \approx 0.3010$, $\log 3 \approx 0.4771$, and $\log 5 \approx 0.6990$, calcuate

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- $\log 7.5 = \log \frac{15}{2} = \log 15 - \log 2 \approx 1.1761 - 0.3010 = 0.8751.$
- $\log 81 = \log 3^4 = 4 \log 3 \approx 4(0.4771) = 1.9084.$
- $\log 50 = \log 5 \cdot 10 = \log 5 + \log 10 \approx 0.6990 + 1 = 1.6990.$

Examples

Expand and simplify the expression $\log_2 \frac{x^2-1}{2^x x^2}$.

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Well,

$$\begin{aligned}
\log_2 \frac{x^2-1}{2^x x^2} &= \log_2(x^2-1) - \log_2(2^x x^2) \\
&= \log_2((x-1)(x+1)) - \log_2(2^x) - \log_2(x^2) \\
&= \log_2(x-1) + \log_2(x-1) - x\log_2 2 - 2\log_2 x \\
&= \log_2(x-1) + \log_2(x+1) - x - 2\log_2 x.
\end{aligned}$$

Example

Write the expression $2\ln x + \frac{1}{2}\ln(x^2 + 1) - x$ as a single logarithm

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Well,

$$\begin{aligned} 2\ln x + \frac{1}{2}\ln(x^2+1) - x &= \ln x^2 + \sqrt{x^2+1} - \ln e^x \\ &= \ln \frac{x^2\sqrt{x^2+1}}{e^x}. \end{aligned}$$

Graphs of Logarithmic Functions

Definition (The Logarithmic Function)

The function defined by $f(x) = \log_b x$ with $b > 0$, $b \neq 1$ is called the logarithmic function with base b. The domain of f is all positive numbers.

If $b < 1$, then the graph of $f(x) = \log_b x$ is

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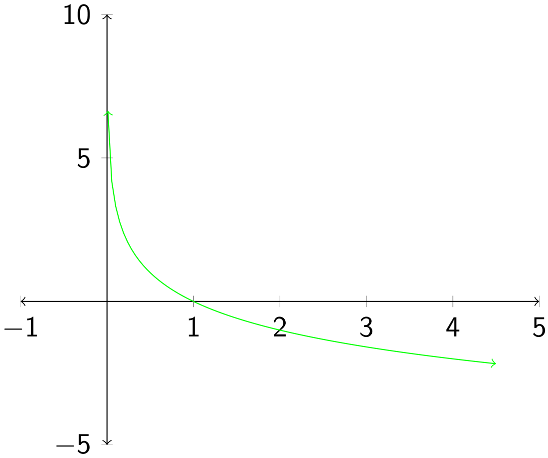
See the graph on the next slide.

Graph of $f(x) = \log_{\frac{1}{2}} x$



Graphs of Logarithmic Functions

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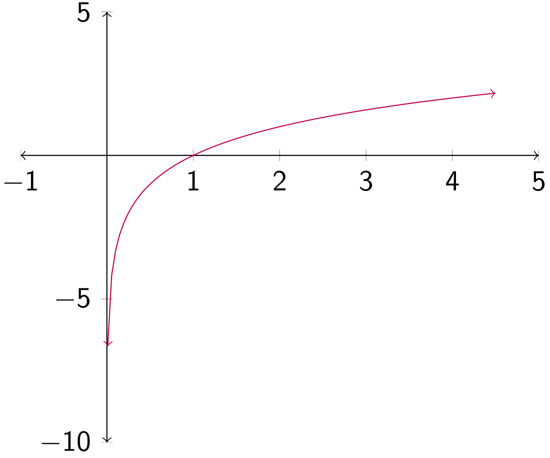
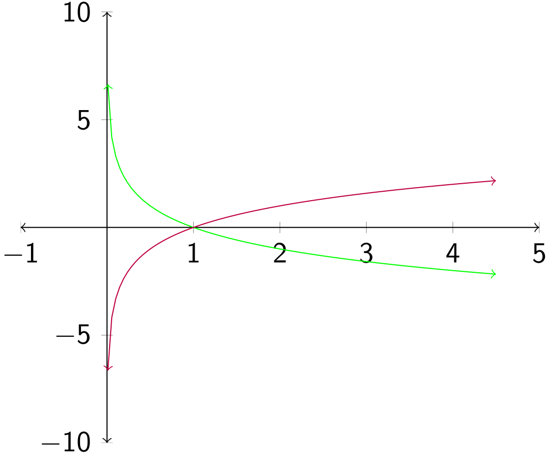
Graph of $f(x) = \log_2 x$



Graphs of $f(x) = \log_{\frac{1}{2}} x$ and $g(x) = \log_2 x$



Change of Base and Relationship Between Logarithmic and Exponential Functions

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When all of the following logarithms are defined, we have

$$\log_b a = \frac{\log_c a}{\log_c b}$$

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Also, the logarithmic and exponential functions are **inverses**!

Lemma (Relationship Between Logarithmic and Exponential Functions)

$\ln e^x = x$ *for all real numbers* x.
$e^{\ln x} = x$ *for all* $x > 0$.

Examples

Solve $5 \ln x + 3 = 0$.

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Well,

$$\begin{aligned} 5\ln x + 3 = 0 &\implies \\ \ln x = -\frac{3}{5} &\implies \\ e^{\ln x} = e^{-\frac{3}{5}} &\implies \\ x = e^{-0.6} &\approx 0.55. \end{aligned}$$

Examples

Solve $2e^{x+2} = 5$.

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Well,

$$\begin{aligned} 2e^{x+2} &= 5 \implies \\ e^{x+2} &= \frac{5}{2} \implies \\ \ln(e^{x+2}) &= \ln\frac{5}{2} \implies \\ x + 2 &= \ln\frac{5}{2} \implies \\ x &= \ln\frac{5}{2} - 2 \approx -1.08. \end{aligned}$$

Assignment

Read 5.3. Do problems 8, 16, 24, 32, 38, 46 in 5.1 and 10, 14, 20, 28, 34, 40, 50 in 5.2.