QMI Lesson 18: Antiderivative and Rules of Integration

C C Moxley

Samford University Brock School of Business

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The Antiderivative

Definition (The Antiderivative)

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The function $F(x) = x^3 + 2x + 1$ is an antiderivative of $f(x) = 3x^2 + 2$ because $F'(x) = 3x^2 + 2 = f(x)$.

Antiderivatives Are Not Unique

Show that $F(x) = \frac{1}{3}x^3 - 2x^2 - x - 1$ and $G(x) = \frac{1}{3}x^3 - 2x^2 - x - 9$ are both antiderivatives of $f(x) = x^2 - 4x - 1$.

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Well,

$$F'(x) = x^2 - 4x - 1 = G'(x) = f(x).$$

Moreover, notice that $F_C(x) = \frac{1}{3}x^3 - 2x^2 - x + C$, where C is just some constant is also an antiderivative of $f(x)$. In fact, this is true in general!

Theorem

Let G be an antiderivative of a function f on an interval I. Then every antiderivative F of f on I must be of the form $F(x) = G(x) + C$, where C is some constant.

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Theorem

Let G be an antiderivative of a function f on an interval I. Then every antiderivative F of f on I must be of the form $F(x) = G(x) + C$, where C is some constant. Also, any function F of this form is an antiderivative of f on I.

Antiderivatives of the Function $f(x) = 2x$

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Well, $F'(x) = 2x + 0 = 2x = f(x)$, so F is an antiderivative of f. Due to the previous theorem, we have that $F(x) + C = x^2 + 1 + C$ is a general formula for antiderivatives of f.

Notice, we could simply rewrite this formula as $F(x) = x^2 + C$. Why? Because the constant term C can be made to "swallow up" all other constant terms.

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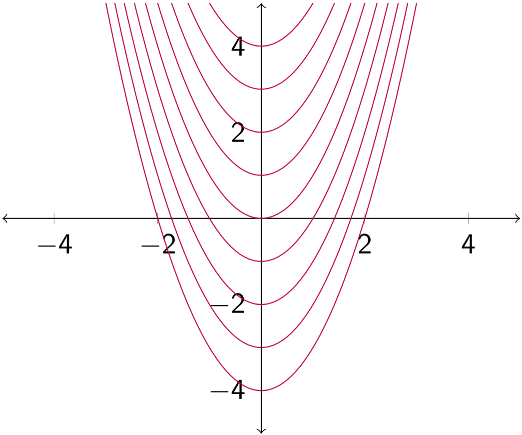

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Using this notation, we can write

$$\int 1 + x \; dx = x + \frac{1}{2}x^2 + C,$$

where C is an arbitrary constant.

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Theorem (The Power Rule)

$$\int x^n \; dx = \frac{1}{n+1} x^{n+1} + C, \qquad n \neq -1$$

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Theorem (Indefinite Integral of a Constant Multiple of a Function)

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Theorem (The Sum Rule)

$$\int f(x) \pm g(x)\ dx = \int f(x)\ dx \pm \int g(x)\ dx$$

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Theorem (The Indefinite Integral of $f(x) = x^{-1}$)

$$\int x^{-1} \; dx = \ln|x| + C, \qquad C \text{ a constant}$$

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For instance, we know that $\int x^2 \ dx = \frac{x^3}{3}$, but if we misapply the rule for the indefinite integral of a constant multiple of a function, we may get

$$\int x^2 \ dx = x^2 \int 1 \ dx = x^2(x) = x^3,$$

which is wrong!

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1. find a general solution and
2. solve for C using the initial condition.

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Let's solve the differential equation from the previous slide where

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so $S(t) = 4t + 3t^{\frac{5}{3}} + 3000$. Therefore,
$S(125) = 4(125) + 3(125^{\frac{5}{3}} + 3000 = 12875.$

Acceleration, Velocity, and Position

A train is accelerating at 2 miles per hour, it's speed at time $t = 0$ is 0 miles per hour, and its initial position at time $t = 1$ is 10 miles down the track. Time is measure in hours. Knowing that the derivative of position gives velocity and that the derivative of velocity gives acceleration, find a formula for the train's position at time t.

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which gives that $s(t) = t^2 + 9$, our particular solution.

Assignment

Read 6.2-6.3. Do problems 4, 8, 20, 36, 38, 50, 54, 58, 62, 70, 100 in 6.1.