QMI Lesson 3:
Modeling with Functions

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Mathematical Models

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4. **Test:** Ensure that your real-world solution is an appropriate answer to your original real-world problem.

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Definition (Linear Function)

A degree one polynomial is a linear function, i.e. it can be written $f(x) = a_1 x + a_0$ with $a_1 \neq 0$.

Example

Overdraft charges are a major source of revenue for some banks. The following gives the revenue from overdraft fees in billions of dollars from 2004 to 2009.

Year (t)	0	1	2	3	4	5
Revenue (y)	27.5	29	31	34	36	38

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Here, t is in years, and $t = 0$ corresponds to year 2004. A linear model giving the approximate projected revenue from overdraft fees is given by

$$f(t) = 2.19t + 27.12 \quad (0 \leq t \leq 5).$$

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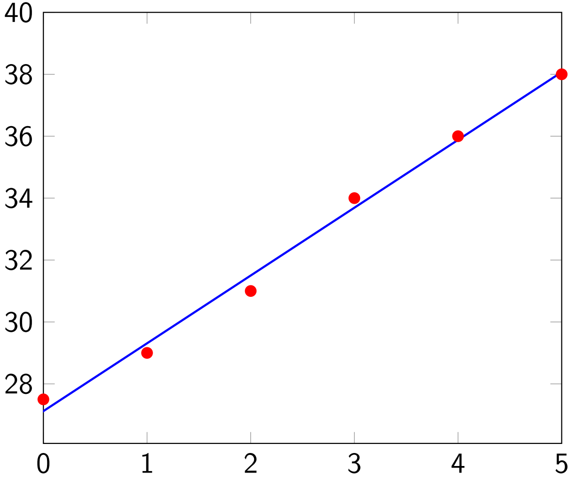

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At what rate did revenue increase from years 2004 to 2009?
The real rate was $\frac{38-27.5}{5} = 2.1$, but the modeled rate was 2.19 (in billions of dollars).

Quadratic Functions

Definition (Quadratic Function)

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The graph of a quadratic function is a parabola which opens up if $a_2 > 0$ or down if $a_2 < 0$.

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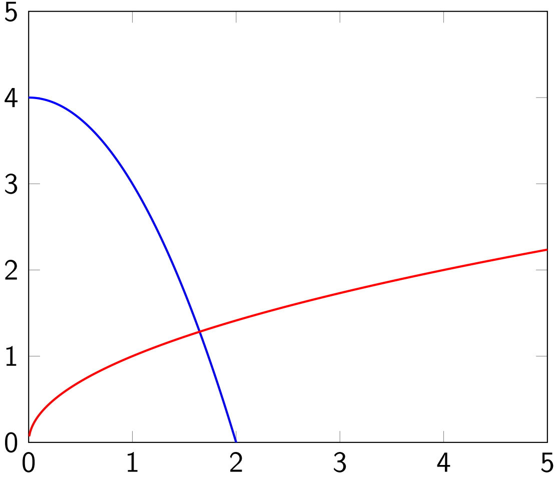
Equilibrium

Definition (Market Equilibrium)

Market equilibrium occurs when supply and demand are equal, i.e. it is when the the supply and demand curves intersect. It corresponds to an equilibrium quantity and price.

Supply, Demand, and Equilibrium



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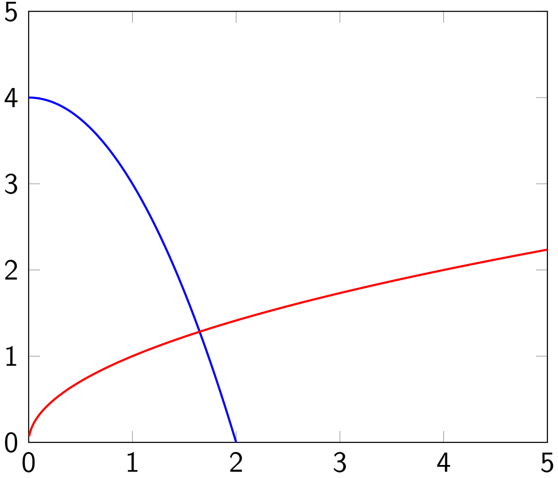

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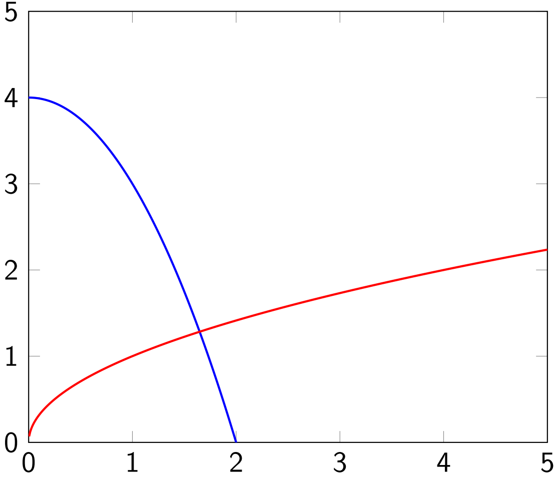

Which curve is the supply curve? The demand curve?

Example

The demand function for a certain Bluetooth device is given by $p = d(x) = -0.025x^2 - 0.5x + 60$, and its supply function is $p = s(x) = 0.02x^2 + 0.6x + 20$, where p is expressed in dollars and x is measured in the thousands of units. Find the equilibrium price and quantity.

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We must solve the system of equations

$$\begin{aligned} p &= -0.025x^2 - 05x + 60 \\ p &= 0.02x^2 + 0.6x + 20. \end{aligned}$$

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Thus, $p = s(20) = d(20) = -0.025(20)^2 - 0.5(20) + 60 = 40$.
Thus, the equilibrium price is \$40 per headset.

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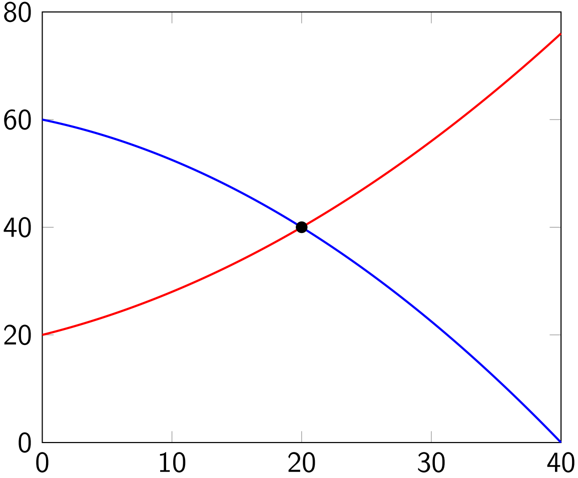

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1. Label each variable in the problem and draw a figure or diagram.
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3. Write a function for the quantity sought. Pay careful attention to the domain of this function.

Example

PlaneX charges \$300 per person if exactly 200 people sign of for a group flight. If more than sign up, the per-person fare is reduced by \$1 for each person. Say x people (where $x \geq 200$) people booked a group flight. Write a function for the total revenue realized by PlaneX when x people booked a group flight.

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Domain $= [200, 500]$.

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- Any polynomial function can be written as a sum of constant multiples of power functions. **True!**
- If $r > 0$, then the power function $f(x) = x^r$ is defined for all values of x. **False.** Take, for instance $r = 1/2$.

Assignment

Read 2.4. Do problems 6, 12, 18, 26, 38, 48, 62, 66, 74, 80, 82 in 2.3.