QMI Lesson 7: Some Differentiation Rules

C C Moxley

Samford University Brock School of Business

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First, we'll go over each rule and its proof. Then we'll move on to examples.

The Derivative of a Constant

Theorem (The Derivative of a Constant)

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$$\frac{d}{dx}[c] = 0.$$

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$$\lim_{h \to 0} \frac{(x^n + nx^{n-1}h + \frac{n(n-1)x^{n-2}h^2}{2!} + \cdots + nxh^{n-1} + h^n) - x^n}{h} =$$

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$$\lim_{h\to 0}[nx^{n-1} + \frac{n(n-1)x^{n-2}h}{2!} + \cdots + nxh^{n-2} + h^{n-1}] = nx^{n-1}$$

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Proof: $\frac{(f/g)(x-h)-(f/g)(x)}{h} = \frac{1}{g(x+h)(g(x)} \cdot \frac{f(x+h)g(x)-f(x)g(x+h)}{h} =$
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$\frac{1}{g(x+h)(g(x)} \cdot [g(x)\frac{f(x+h)-f(x)}{h} - f(x)\frac{g(x+h)-g(x)}{h}]$. And passing through the limit on both sides of the equality proves the theorem.

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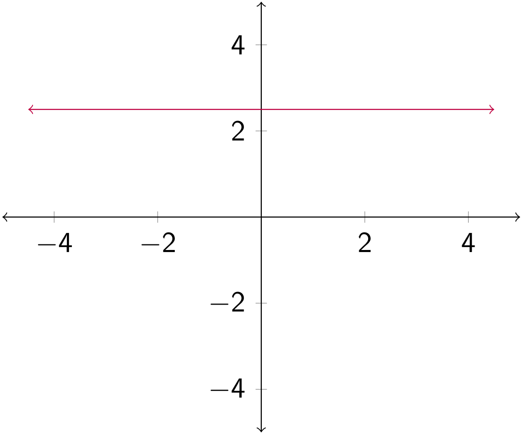

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- $f(x) = x \implies f'(x) = 1 \cdot x^{1-1} = 1x^0 = 1.$
- $g(h) = \frac{1}{h^\pi} \implies$

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What is the rate of change in the population after 2 months? 6 months? What is the final population after implementation of the breeding program?

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  $\frac{1+5x^3}{2\sqrt{x}(x^3-1)^2}$.

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For instance, if we used this incorrect rule, we would calculate for $h(x) = \frac{x}{x^2+1}$ that

$$h'(x) = \frac{1}{2x},$$

but we know this is wrong since we already calculated

$$h(x) = \frac{1-x^2}{(x^2+1)^2}.$$

Assignment

Read 3.3-3.4. Do problems 28, 31, 36, 38, 42, 50, 70, 78 in 3.1 and 12, 26, 42, 62, 68 in 3.2.