Practice Test 1 Solutions
BUSA130

This exam is graded out of 100 points. Show all work necessary to solve the problems. You have 65 minutes.

1) Below are the solutions to Problem 1 in order.

- $\lim\limits_{h\to 0}\dfrac{2-\sqrt{4-h}}{h} = \lim\limits_{h\to 0}\left[\left(\dfrac{2-\sqrt{4-h}}{h}\right)\left(\dfrac{2+\sqrt{4-h}}{2+\sqrt{4-h}}\right)\right] = \lim\limits_{h\to 0}\dfrac{4-4+h}{h(2+\sqrt{4-h})} =$
  $\lim\limits_{h\to 0}\dfrac{h}{h(2+\sqrt{4-h})} = \lim\limits_{h\to 0}\dfrac{1}{2+\sqrt{4-h}} = \dfrac{1}{4}$.

- $\lim\limits_{x\to\infty}\dfrac{-2x^2+x}{1-x^2} = 2$ because the degrees are the same and 2 is the ratio of the leading coefficients.

- $\lim\limits_{x\to\infty}\dfrac{x^2-1}{x-1} =$ **DNE** because the degree on top is larger than the degree on the bottom.

2) It defines a function from x to y because it passes the vertical line test, but it does not define a function from y to x because it fails the horizontal line test.

3) Below are the solutions to Problem 3 in order.

- We need $x+1 \geq 0$ and $2x^2 - 12x + 10 \neq 0$, so

  $$x+1 \geq 0 \implies x \geq -1$$

  and also

  $$2x^2 - 12x + 10 \neq 0 \implies x^2 - 6x + 5 \neq 0 \implies (x-5)(x-1) \neq 0 \implies x \neq 1, 5.$$

  So, our domain is $[-1, 1) \cup (1, 5) \cup (5, \infty)$.

- We need $x - 2 \neq 0$ and $\frac{x}{x-2} \geq 0$. The first condition tells us that $x \neq 2$ the second tells us to break the number line over 0 and 2 and to test the intervals $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$. Testing them (in that order) yields true, false, true. Thus, we must exclude the middle interval. We also must exclude the number 2. But we do not exclude 0 because it does not result in a zero in the denominator or a negative value under the square root. So our domain is $(-\infty, 0] \cup (2, \infty)$.

4) The function is not differentiable at $x = 1$ because the tangent line is vertical. It is not differentiable at $x = 0$ because of the sharp turn. It is not differentiable at $x = -1$ because of the discontinuity.

5) The four-step method is given below.

1. $f(x+h) = \dfrac{-2}{x+h}$.
2. $f(x+h) - f(x) = \dfrac{-2}{x+h} + \dfrac{2}{x} = \dfrac{-2(x)}{(x+h)x} + \dfrac{2(x+h)}{(x+h)x} = \dfrac{2h}{(x+h)(x)}$.
3. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2}{(x+h)x}$.
4. $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} \dfrac{2}{(x+h)x} = \dfrac{2}{x^2}$.

Thus, $f'(x) = \frac{2}{x^2}$, and the slope of the tangent line at $x = 2$ is $\frac{1}{2}$. Therefore, the equation of the tangent line at the point $(2, -1)$ is $y + 1 = \frac{1}{2}(x - 2)$.

6) Below are the solutions to Problem 6 in order.

- False! As a counterexample, consider $f(x) = 2$ and $a = 0$. Then if we let $g(x)$ be the piecewise defined function where $g(x) = 0$ if $x \neq 0$ and $g(0) = 10$. Then we have that $\lim\limits_{x \to 0} f(x) = 2$ and $g(0) = 10$, but $\lim\limits_{x \to 0} [f(x)g(x)] = 0 \neq 20$.
- True! The equation of a linear function is $f(x) = mx + b$, but if the function contains the point $(0, 0)$, then b must be 0 - to see this, just plug in $(0, 0)$ to the equation. Thus the function must take the form $f(x) = mx$. So $f(ax) = m(ax) = a(mx) = a \cdot f(x)$, as the statement says. (Remember, you do not have a provide any reasoning for a true statement.)
- False! As a counterexample, provide two points on the graph of the equation that have the same x value but different y values, like $(0, -1)$ and $(0, 1)$.

7) We use the equation describing revenue $R = px$ where x is the number of loaves sold and p is the price per loaf. Well, $p = 1.02 - 0.005(x - 100)$ because for each loaf exceeding 100, we take of \$0.005 from the price \$1.02. Simplifying, we get $p = 1.52 - 0.005x$. Thus, $R(x) = (1.52 - 0.005x)x = 1.52x - 0.005x^2$. The domain of this function is $[100, 304]$. This domain comes from the fact that we assume the number of loaves purchased was at least 100 and that revenue should be positive.

8) We would like to find a and b so that we can calculate $d(91125)$. We know that $d(64000) = 1$ and $d(42875) = 1.5$. This gives the system of equations

$$1 = 40a + b \qquad \text{and} \qquad 1.5 = 35a + b.$$

Solving the first for b, we get $b = 1 - 40a$, so if we plug this expression for b into the second equation, we get

$$1.5 = 35a + (1 - 40a) \implies 1.5 = -5a + 1 \implies -0.5 = -5a \implies a = -0.1.$$

This means that $a = -0.1$ and we plug this value for a into the equation $b = 1 - 40a$, which gives $b = 1 - 40(-0.1) = 5$. This means that

$$d(x) = -0.1\sqrt[3]{x} + 5,$$

and so $d(91125) = 0.5$, which means that a market price per download of \$0.50 would support a demand of 91,125 downloads.