Practice Test 1 Solutions
BUSA130

This exam is graded out of 100 points. Show all work necessary to solve the problems. You have 65 minutes.

1) Well, we know that $f(20) = 0$, which means $-300(20) + I = 0 \implies I = 6000$. Thus, $f(x) = -300x + 6000$ and $f(6) = 4200$, meaning that the roaster is worth \$4200 after 6 years.

2) The correct answers are (b), (c), and (d).

3) We need to solve the system of equations given. We simply set them equal to one another since we want $p_1 = p_2$. Thus,

$$300-x^2 = 5x+150 \implies 0 = x^2+5x-150 \implies 0 = (x+15)(x-10) \implies x = -15, 10.$$

Thus, the market equilibrium quantity is 10 whereas the market equilibrium price is $p_1(10) = p_2(10) = 200$. Also, p_1 is the demand equation (since it is decreasing on $x \geq 0$) and p_2 is the supply equation (since it is increasing on $x \geq 0$).

4) Below are the solutions to Problem 4 in order.

- $\lim\limits_{x\to 9} \dfrac{x^2-81}{\sqrt{x}-3} = \lim\limits_{h\to 0} \left[\left(\dfrac{x^2-81}{\sqrt{x}-3}\right)\left(\dfrac{\sqrt{x}+3}{\sqrt{x}+3}\right)\right] = \lim\limits_{x\to 9} \dfrac{(x^2-81)(\sqrt{x}+3)}{x-9} =$ $\lim\limits_{x\to 9} \dfrac{(x-9)(x+9)(\sqrt{x}+3)}{x-9} = \lim\limits_{x\to 9} [(x+9)(\sqrt{x}+3)] = 108.$
- $\lim\limits_{x\to -\infty} \dfrac{3x^4-5x^2+2}{1-x^5} = 0$ because the degree on the bottom is higher than the degree on the top.
- $\lim\limits_{x\to 1} \dfrac{x^2+2x+1}{x^2-1} = \lim\limits_{x\to 1} \dfrac{(x+1)^2}{(x+1)(x-1)} = \lim\limits_{x\to 1} \dfrac{x+1}{x-1} =$**DNE**. The expression does not simplify to allow us to plug in. You can see that the limit does not exist by plugging in numbers closer and closer to 1 on the left- and right-hand sides. The left-hand limit goes to negative infinity and the right hand limit goes to positive infinity, so the limit does not exist. You could also see the limit does not exist because there is a vertical asymptote at $x = 1$ on the graph of the function.

- $\lim_{x \to 1} \sqrt{\frac{x^2+2x+1}{x}} = 2$. For this limit, simply plug in.

5) Parts (a), (b), and (d) are all true. Part (c) is false, and there are multiple ways to correct the statement. When you correct the statement, only change one side of the equality. Here are a few possibilities for a corrected (c):

- $\lim_{x \to -1^+} f(x) = 0$.
- $\lim_{x \to -1} f(x) = \textbf{DNE}$.
- $\lim_{x \to 1} f(x) = 0$.

6) Part (a) is true. Part (b) is false. Take as a counterexample the function $f(x) = x^3$, which is a power function but for which $f(ax) = (ax)^3 = a^3x^3 = a^3 \cdot f(x) \neq a^2 \cdot f(x)$.

7) The four-step method is given below.

1. $f(x+h) = (x+h+1)^2 = (x+h)^2 + 2(x+h) + 1 = x^2 + h^2 + 2xh + 2x + 2h + 1$.
2. $f(x+h) - f(x) = (x^2 + h^2 + 2xh + 2x + 2h + 1) - (x^2 + 2x + 1) = h^2 + 2xh + 2h$.
3. $\frac{f(x+h) - f(x)}{h} = h + 2x + 2$.
4. $\lim_{h \to 0}(h + 2x + 2) = 2x + 2 = 2(x+1)$.

Thus, $f'(x) = 2(x+1)$, and the slope of the tangent line at $x = 0$ is 2. Therefore, the equation of the tangent line at the point $(0, 1)$ is $y - 1 = 2(x - 0)$.

8) Below are the solutions to Problem 8 in order.

- We need $x - 1 \geq 0$ and $x^2 - 11x + 30 \neq 0$, which means that x cannot equal 5 or 6 and must be greater than or equal to 1. This means we have the domain $[1, 5) \cup (5, 6) \cup (6, \infty)$.
- From the denominator, we see that x cannot be -2. Breaking the number line over the zeros of numerator and denominator, we see that we must test the statement $\frac{x^2-1}{3x+6} \geq 0$ on the intervals $(-\infty, -2)$, $(-2, -1)$, $(-1, 1)$, and $(1, \infty)$. We get (in order) false, true, false, true. We include 1 and -1 but exclude -2 (because it creates a zero in the denominator). Thus, our domain is $(-2, -1] \cup [1, \infty)$.