Practice Test 2 Solutions
BUSA130

This exam is graded out of 15 points. You must do Problem 1, and you may choose 6 other problems from Problems 2, 3, 4, 5, 6, 7, 8. Do not do all eight problems! Show all work necessary to solve the problems unless otherwise instructed. You have 65 minutes.

1) For the first function, we have

$$f'(x) = 3\left(\frac{3x^2-x}{-x^2-5x}\right)^2 \cdot \frac{d}{dx}\left(\frac{3x^2-x}{-x^2-5x}\right)$$

$$= 3\left(\frac{3x^2-x}{-x^2-5x}\right)^2 \cdot \left(\frac{(-x^2-5x)(6x-1)-(3x^2-x)(-2x-5)}{(-x^2-5x)^2}\right)$$

$$= 3\left(\frac{3x^2-x}{-x^2-5x}\right)^2 \left(\frac{-16x^2}{(-x^2-5x)^2}\right).$$

For the second function, we have

$$g'(x) = 3\left(2-\frac{1}{2\sqrt{x}}\right)\sqrt{x^5-3x+1} + \frac{3}{2}(2x-\sqrt{x})\frac{1}{\sqrt{x^5-3x-1}}(5x^4-3).$$

For the third function, we have

$$h'(d) = (-1)(d-3)^{-2}(1) + 10d = -\frac{1}{(d-3)^2} + 10d.$$

2) Well,

$$\frac{d}{dx}\left[(f\circ h)(x)\cdot f(x)\right] = \left[f'(h(x))\cdot h'(x)\right]\cdot f(x) + f(h(x))\cdot f'(x),$$

therefore we get

$$\frac{d}{dx}\left[(f\circ h)(1)\cdot f(1)\right] = \left[f'(h(1))\cdot h'(1)\right]\cdot f(1) + f(h(1))\cdot f'(1) =$$

$$[f'(-1)\cdot 4]\cdot(-2)+f(-1)\cdot 2=\frac{1}{2}4\cdot(-2)+3\cdot 2=-4+6=2.$$

3) $P(2.5)\approx 42.24\%$ without children in 1985. $P(4)\approx 37.21\%$ without children in 2000. And $P'(3.5)=-0.27(54.1)(3.5)^{-1.27}\approx -2.98\%$ per decade.

4) Statement 1: False. Counterexample: $h(x)=x^2$ and $f(x)=x+1$. Then $g(x)=h(f(x))=x^2+2x+1$, so $g'(x)=2x+2\neq(h'\circ f')(x)$ since $h'(x)=2x$ and $f'(x)=1$, so $(h'\circ f')(x)=2(1)=2\neq 2x+2$.
Statement 2: False! We can see this is false if we let $\overline{P}'(10)=-0.01$ but let $\overline{P}(10)=10$. Then $\overline{P}(11)$ would be about 9.9. So the total profit would be \$100 at level 10 units but \$108.90 at level 11 units, so profit has increased.

5) Solve for $x=f(p)$, getting

$$f(p)=x=50-1.25p \implies f'(p)=-1.25.$$

Thus, $E(p)=\dfrac{-p(-1.25)}{50-1.25p}\implies E(10)=\dfrac{12.5}{36.5}=\dfrac{1}{3}<0$. So demand is inelastic at $p=10$.

6) Well, $f'(x)=g'(x^2)\cdot 2x$, so

$$f''(x)=g''(x^2)(2x)(2x)+g'(x^2)2=4x^2g''(x^2)+2g'(x^2).$$

7) First, recognize that p and x are actually functions with respect to t. Differentiate both side with respect to t to get

$$p'(t)-\frac{1}{4}x(t)x'(t)=0.$$

Then, plugging in known values we get

$$p'(t)-\frac{1}{4}(20)(1)=0.$$

Thus, $p'(t)=5$, meaning that the price per pedicure is increasing at a rate of \$5 per day.

8) Let $f(x) = \sqrt[5]{x}$, then we need to calculate $f(32.2)$, which is approximated by the differential where $x = 32$ and $dx = 0.2$. We get

$$f(32.2) \approx f(32) + f'(32)(0.2) = 2 + \frac{1}{5}\left(32^{-\frac{4}{5}}\right)(0.2) = 2.0025.$$