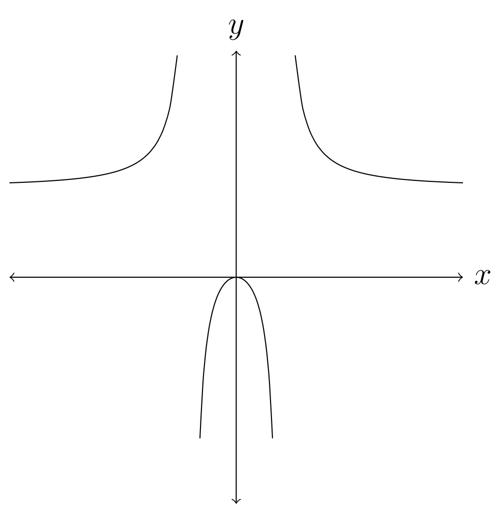
Practice Test 3 Solutions
BUSA130

Part I

1) The domain is everywhere the denominator is not equal to zero, so it is $D = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$. To get the y-intercept, we plug in 0 into the function to get the point $(0, 0)$. To get the x-intercept, we solve the equation $0 = f(x)$ for x, giving the point $(0, 0)$. (Notice, our x- and y-intercepts are the same in this case.) The vertical asymptotes occur where the denominator is 0 while the numerator is not, so the lines $x = 2$ and $x = -2$ are vertical asymptotes. Our horizontal asymptote is found by calculating $\lim\limits_{x \to \pm\infty} f(x) = 4$, so the line $y = 4$ is a horizontal asymptote. To find the relative extrema, we need the first derivative. $f'(x) = -\frac{32x}{(x^2-4)^2}$. We must test the intervals $(-\infty, -2)$, $(-2, 0)$, $(0, 2)$, $(2, \infty)$, getting that the function is increasing, increasing, decreasing, and decreasing on those intervals respectively. Thus, there is a local maximum at $(0, 0)$. We need to second derivative as well: $f''(x) = \frac{96x^2+128}{(x^2-4)^3}$, which means we must test the intervals $(-\infty, -2)$, $(-2, 2)$, $(2, \infty)$, getting that the function is concave up, down, and up respectively. However, there are no inflection points because the function is not defined at $x = \pm 2$. Using this information, we may produce the graph.



2) We solve the problems separately.

- $4^{4x-2} = 16^{x^2} \implies 2^{2(4x-2)} = 2^{4(x^2)} \implies 8x - 4 = 4x^2 \implies 0 = 4x^2 - 8x + 4 \implies x = 1.$
- $\log_x 100 = -\frac{1}{2} \implies x^{-\frac{1}{2}} = 100 \implies x = 100^{-2} \implies x = \frac{1}{10000}.$
- $\log 10^x - \ln e = 0 \implies x - 1 = 0 \implies x = 1.$

Part II

1) We do these separately.

- If you graph $f(x) = |x-2| + 1$, you see that there is a relative minimum at $x = 2$. And the relative minimum is (2, 1). You could also conduct a first derivative test and see that the first derivative is undefined at $x = 2$, so it is a critical number, and you'd test the first derivative on the number line broken over this point.

- We need the first derivative of $g(x) = x^3 - 3x$, which is $g'(x) = 3x^2 - 3$. Thus, our critical numbers are $x = \pm 1$, breaking the number line over these points, we get that the function is increasing on $(-\infty, -1)$, decreasing on $(-1, 1)$, and increasing on $(1, \infty)$. Thus, there is a local maximum at $x = -1$ which is (-1,2) and a local minimum at $x = 1$ which is (1,-2).

2) We need the second derivative of $f(x) = -3x^4 - 4x^3$, which is $f''(x) = -36x^2 - 24x = -12x(3x+2)$, so we use the second derivative to test the number line broken over the points 0 and $-\frac{2}{3}$, getting that the function is concave down on $(-\infty, -\frac{2}{3})$, concave up on $(-\frac{2}{3}, 0)$, and concave down on $(0, \infty)$. So we have inflection points at $x = 0$ and $x = \frac{2}{3}$. They are (0,0) and $(-\frac{2}{3}, \frac{16}{27})$.

3) We want to minimize the total cost function (per mile) which is comprised of the cost for fuel (per mile) and the cost for labor (per mile). The cost for fuel is given by the amount of fuel we use times the cost of fuel. If we get $\frac{20}{x}$ miles per gallon traveling at x miles per hour and if fuel costs 4 dollars per gallon, we use $\frac{x}{5}$ dollars on fuel for ever mile we operate the ferry. If the captain is paid \$20 per hour and we travel at x miles per hour, then we spend $\frac{20}{x}$ dollars per mile on labor. Thus, our total cost is

$$C(x) = \frac{x}{5} + \frac{20}{x}.$$

We consider the function on the interval [6,15] because of the setup of our problem. Differentiating, we get $C'(x) = \frac{1}{5} - \frac{20}{x^2} = \frac{x^2-100}{5x^2}$. Thus, our critical values are $x = \pm 10$, so we test $f(6)$, $f(10)$, and $f(15)$, which shows that $x = 10$ minimizes the cost. The minimal cost rate is \$4 per mile.

4) We get $\ln\left[\dfrac{(e^{2x}10)}{\sqrt{x}}\right] = \ln(e^{2x}) + \ln 10 - \ln\sqrt{x} = 2x\ln e + \ln 10 - \dfrac{1}{2}\ln x = 2x + \ln 10 - \dfrac{1}{2}\ln x$.

5) Well, $f'(x) = 6(3x+1)$, so our critical value is $-\frac{1}{3}$, and we test $f(-2)$, $f(-\frac{1}{3})$, and $f(3)$ and get that $f(3) = 100$ is our absolute maximum on the interval.

6) The solid function is the second derivative of the dotted function because the dotted function is concave down everywhere and the solid function is negative everywhere.