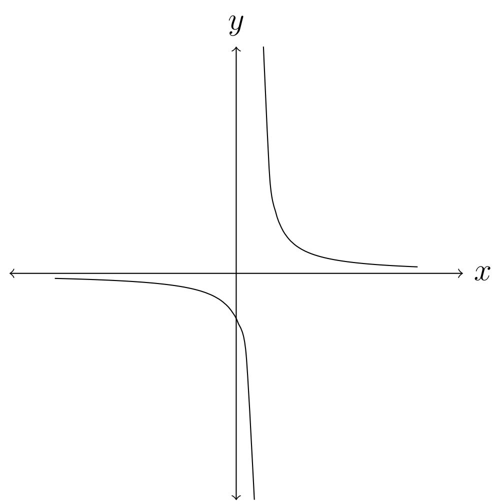
Practice Test 3 Solutions
BUSA130

Part I

1) The domain is everywhere the denominator is not equal to zero, so it is $D = (-\infty, 1) \cup (1, \infty)$. To get the y-intercept, we plug in 0 into the function to get the point $(0, -2)$. To get the x-intercept, we solve the equation $0 = f(x)$, but this cannot be solved. There is no x-intercept. The vertical asymptotes occur where the denominator is 0 while the numerator is not, so the line $x = 1$ is the vertical asymptote. Our horizontal asymptote is found by calculating $\lim_{x \to \pm\infty} f(x) = 0$, so the line $y = 0$ is a horizontal asymptote. To find the relative extrema, we need the first derivative. $f'(x) = -\frac{2}{(1-x)^2}$. We must test the intervals $(-\infty, 1)$, $(1, \infty)$, getting that the function is decreasing and decreasing on those intervals respectively. Thus, there are no local extrema. We need to second derivative as well: $f''(x) = -\frac{4}{(1-x)^3}$, which means we must test the intervals $(-\infty, 1)$, $(1, \infty)$, getting that the function is concave down and up respectively. However, there are no inflection points because the function is not defined at $x = 1$. Using this information, we may produce the graph.



2) Solve the equations below. (1pt each)

- $\left(\frac{1}{4}\right)^{2-4x} = 4^{2x^2} \implies 2^{-2(2-4x)} = 2^{2(2x^2)} \implies -4+8x = 4x^2 \implies 4x^2 - 8x + 4 = 0 \implies x^2 - 2x + 1 = 0 \implies (x-1)^2 = 0 \implies x = 1.$
- $\log_x x = 2 \implies x^2 = x \implies x^2 - x = 0 \implies x(x-1) = 0 \implies x = 0, 1.$
- $\log 100^x - \ln 1^2 = 0 \implies x \log 100 - 2 \ln 1 = 0 \implies 2x - 0 = 0 \implies x = 0.$

Part II

1) Using the table, we see that the rate of increase decreases as it passes over Day 4, so our inflection point is (4,83). For a new trainee, she can pack an increasing number of boxes every day at an increasing rate for the first four days, after which the rate of increase in the number of boxes packed per day begins to decrease.

2) We get $f'(x) = 2x + 1$, and our only critical point is $x = -\frac{1}{2}$. Testing the real number line broken over this point, we see that this corresponds to a local minimum. The local minimum is $\frac{3}{4}$. This is a global minimum because the function is always concave up (since the second derivative is $f''(x) = 2$).

3) If p is the number of peaches per tree and d is the number of trees per acre, then the total yield Y is given by $Y = 10pd$. Because we have the point (200,30) and the slope $-\frac{1}{5}$, we get the constraint equation $d - 30 = -\frac{1}{5}(p - 200) \implies d = -\frac{1}{5}p + 70$. This gives $Y(p) = 10p(-\frac{1}{5}p + 70) = -2p^2 + 700p$, and so $Y'(p) = -4p + 700$, so our critical value is 175. Now! The interval on which we must consider Y begins at 0 (because that's a natural starting place for the number of peaches per tree). But because d must be positive and because $d = -\frac{1}{5}p + 70$, we get that $0 \le -\frac{1}{5}p + 70 \implies p \le 350$. So we test $Y(0)$, $Y(175)$, $Y(350)$ and see that the maximum yield occurs at $p = 175$ (which means $d = 35$). Here, the maximum yield is 61250 peaches.

4) $12 - e^{-0.4x} = 3 \implies e^{-0.4x} = 9 \implies -0.4x = \ln 9 \implies x = -\frac{5 \ln 9}{2} \approx -5.49.$

5) The solid function is the derivative of the dotted function because the dotted function is increasing where the solid function is positive and is decreasing where the solid function is negative.