Lecture 4:
Chapter 4

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UAB Mathematics

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§4.2 Basic Concepts of Probability

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three tests	PPP or FFF	PFP	$\{\text{PPP}, \text{PPF}, \ldots, \text{FFF}\}$

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- **Subjective probabilities:** Estimate $P(A)$ by using knowledge of relevant circumstances.

§4.2 Examples

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$$\frac{205}{1010} \approx 20.3\%$$

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Use the subjective probability method: Only 1 in 100000 people in the US own both a cane and tophat. The probability is very small, maybe 0.00001.

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Use the classical approach. There are 52^2 possible outcomes, of which 4^2 are drawing a king from the first deck and a king from the second deck. So, the probability is $\frac{16}{2704} \approx 0.59\%$.

§4.2 Definitions

Definition (Complement of an Event)

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Definition (Unusual/Unlikely)

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Discuss the rare event rule and how it is used to investigate hypotheses.

§4.3 Addition Rule

If you want to determine the chances of an outcome being in event A or event B, use the addition rule:

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What is the chance that a randomly selected member of this sample had either contacts or glasses?

$$P(\text{contacts or glasses}) = \frac{30}{50} + \frac{35}{50} - \frac{23}{50} = \frac{42}{50}.$$

§4.3 Complementary Events Rule

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Example (Find $P(\overline{A \text{ or } B})$.)

$P(\overline{A \text{ or } B}) = 1 - P(A) - P(B) + P(A \text{ and } B).$

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Discuss $P(A \text{ and } B)$ and $P(B|A)$.

The probability that two events occur is equal to the probability that the first occurs times the probability that the second occurs **if these events are independent**!

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$P(A \text{ and } B) = P(A)P(B)$	$P(A \text{ and } B) = P(A)P(B\|A)$

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What is the probability that two randomly selected grades are both Bs?

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What is the probability that two randomly selected grades are both Bs? With replacement: $\frac{9}{25}$. Without replacement: $\frac{87}{245}$

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What is the probability that two randomly selected grades are both Bs? With replacement: $\frac{9}{25}$. Without replacement: $\frac{87}{245}$

Example (What are the chances that 26 randomly chosen people have all different birthdays?)

$$\frac{(365)(364)(363)\ldots(341)(340)}{(365)(365)(365)\ldots(365)(365)} = \frac{(365)(364)(363)\ldots(341)(340)}{365^{26}} \approx 40.18\%$$

§4.4 Multiplication Rule: Redundancy

The multiplication rule for independent events helps illustrate why some important industrial components have redundancy: If an oil pipeline has five different oil pressure measuring tools to ensure that the pipeline is not leaking oil and if each of these tools has a fail rate of 5%, then what is the probability that oil is leaking without being detected?

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$$0.05^5 = 0.0000003125.$$

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- First, compute $P(\bar{A})$.
- Then subtract it from 1 because $P(A) = 1 - P(\bar{A})$.

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Now, if we wanted to compute the probability that at least one birthday is shared amongst 26 people (which we will call event A), we can calculate

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$$1 - 0.4018 = 0.5982.$$

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	Positive Test	Negative Test
TB+	15	3
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$$P(A|B) = \frac{55}{505} = 0.1089.$$

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$$P(A|B) = \frac{450}{453} = 0.9934.$$

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Warning: $P(A|B) \neq P(B|A)$!

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5. $\frac{n!}{(n-r)!r!} =$ number of different combinations of r items chosen without replacement from n different items.

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5. How many different ways are there of chosing four letters from the first six letters of the alphabet if order doesn't matter? $\frac{6!}{4!2!} = 15$.