# Lecture 4: Chapter 4

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**UAB Mathematics** 

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# §4.2 Basic Concepts of Probability

Procedure | Event | Simple Event | Sample Space

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rolling a die	6 or 2	6	{1,2,3,4,5,6}
three tests	PPP or FFF	PFP	$\{PPP, PPF, \dots, FFF\}$

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**Subjective probabilities:** Estimate P(A) by using knowledge of relevant circumstances.



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$$\frac{205}{1010} \approx 20.3\%$$

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Use the subjective probability method: Only 1 in 100000 people in the US own both a cane and tophat. The probability is very small, maybe 0.00001.

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Use the classical approach. There are  $52^2$  possible outcomes, of which  $4^2$  are drawing a king from the first deck and a king from the second deck. So, the probability is  $\frac{16}{2704} \approx 0.59\%$ .

#### §4.2 Definitions

#### Definition (Complement of an Event)

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Discuss the rare event rule and how it is used to investigate hypotheses.

If you want to determine the chances of an outcome being in event A or event B, use the addition rule:

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What is the chance that a randomly selected member of this sample had either contacts or glasses?



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What is the chance that a randomly selected member of this sample had either contacts or glasses?

$$P(\text{contacts or glasses}) = \frac{30}{50} + \frac{35}{50} - \frac{23}{50} = \frac{42}{50}.$$



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## Example (Find $P(\overline{A} \text{ or } B)$ .)

$$P(\overline{A \text{ or } B}) = 1 - P(A) - P(B) + P(A \text{ and } B).$$

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Discuss P(A and B) and P(B|A).

The probability that two events occur is equal to the probability that the first occurs times the probability that the second occurs **if these events are independent**!

Independent	Dependent
P(A  and  B) = P(A)P(B)	P(A  and  B) = P(A)P(B A)

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Example (50 Tests: 10 As, 30 Bs, 5 Cs, 5 Ds)

What is the probability that two randomly selected grades are both Bs?

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Example (What are the chances that 26 randomly chosen people have all different birthdays?)

$$\frac{(365)(364)(363)...(341)(340)}{(365)(365)(365)(365)(365)(365)} = \frac{(365)(364)(363)...(341)(340)}{365^{26}} \approx 40.18\%$$



# §4.4 Multiplication Rule: Redundancy

The multiplication rule for independent events helps illustrate why some important industrial components have redundancy: If an oil pipeline has five different oil pressure measuring tools to ensure that the pipeline is not leaking oil and if each of these tools has a fail rate of 5%, then what is the probability that oil is leaking without being detected?

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$$0.05^5 = 0.0000003125.$$

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- First, compute  $P(\bar{A})$ .
- Then subtract it from 1 because  $P(A) = 1 P(\bar{A})$ .

Now, if we wanted to compute the probability that at least one birthday is shared amongst 26 people (which we will call event A), we can calculate

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$$1 - 0.4018 = 0.5982.$$

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TB+	15	3
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$$P(A|B) = \frac{450}{453} = 0.9934.$$

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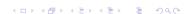
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Warning:  $P(A|B) \neq P(B|A)!$ 



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- 5 How many different ways are there of chosing four letters from the first six letters of the alphabet if order doesn't matter?  $\frac{6!}{4!2!} = 15$ .