# Lecture 7:
# Chapter 7

C C Moxley

UAB Mathematics

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# §7.1 Inferential Statistics

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**Definition (Inferential Statistics)**

The use of sample data to make statements about the statistical/probabilistic characteristics of a population.

# §7.2 Estimating a Population Proportion

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E.g. 35% (or 0.35) is a point estimate.

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E.g. 95% (or 0.95) is a confidence level. In this case, $\alpha = 0.05$. Often, you are asked to create the confidence interval given a certain desired confidence level. In this chapter, we use confidence intervals for informal hypothesis testing.

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## Definition (Critical Value)

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**Different distributions must have critical values calculated differently!**

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## Definition (Margin of Error)

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For proportions, we have

$$|p - \hat{p}| \leq E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}.$$

You can also use this information to calculate how big a sample size needs to be to yield a certain level of confidence with a certain error size. (Always round up when doing this.)

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which implies that $n \approx 3269.78$, so we need a sample size of at least 3270.

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- The conditions for a binomial distribution are satisfied.
- There are at least five successes and five failures.

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$$n - 1,$$

where $n$ is the sample size.

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50 body temperatures of cats were taken and their average was $101.5^\circ F$. It's known that cats have body temperatures which have a standard deviation of $0.7^\circ F$. Find a 95% confidence interval for the mean body temperature of cats.

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which means our sample size must be at least 133 cats.

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Note: The $\chi^2$ distribution is **not symmetric**, and neither is the confidence interval it creates. For this reason, we never denote a confidence interval for the standard deviation in the $\sigma \pm E$ notation! We always use interval notation!

# §7.4 The $\chi^2$ Distribution

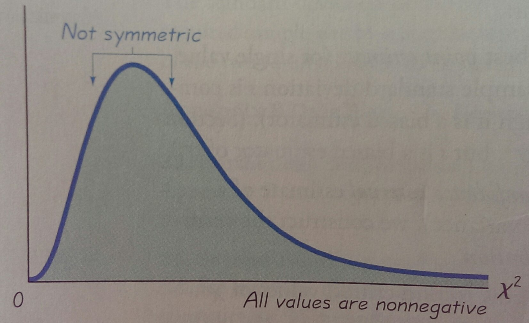


**Figure 7-7** **Chi-Square Distribution**

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$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}},$$

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- The population **must be normally distributed, even if the sample is large!**

# §7.4 Example

The time a student spends on a section of the SAT is given in minutes: 25, 21, 21, 18, 13, 13, 14, 16, 12, 22, 25, 25, 30. These times are known to be normally distributed. Calculate a 90% confidence interval for the standard deviation of these times.

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We have that $\overline{x} = 19.62$ and $s = 5.752$.

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Therefore,

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}} \implies$$

$$4.35 = \sqrt{\frac{(12)5.7524^2}{21.0261}} < \sigma < \sqrt{\frac{(12)5.7524^2}{5.2260}} = 8.72.$$

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This means that if we took a large number of simple random samples of size 13 and calculated their standard deviation, then 90% of the time, your calculated standard deviation would fall into the interval (4.35,8.68).

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This means that if we took a large number of simple random samples of size 13 and calculated their standard deviation, then 90% of the time, your calculated standard deviation would fall into the interval (4.35,8.68). It does not mean that there is a 90% chance that $\sigma$ falls within the interval.

# §7.4 Precise Language

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What does it mean to have a 95% CI for population mean? It is precisely an interval which, when a random selection is made from the sampling distribution of the mean, will contain the given selection 95% of the time. We'll talk about how this relates to hypothesis testing in the next chapter, and it will be much easier to speak about what a 95% CI (or confidence level) means in the context of hypothesis testing.

# §Review

In a probability histogram for the number of female girls in a family of 5, how would you determine the probability of having two or three females?

# §Review

In some population, the mean is 25 and the standard deviation is 6. What is the probability that, in a simple random sample of 36, the sample mean would be between 24 and 26?

# §Review

20% of students at UAB live on campus. Approximate the probability of choosing 10 students at random from the student population and having all of them live on campus.

# §Review

What is the probability of having exactly one student who does not live on campus under the same circumstances as in the previous question?

# §Review

What is the $z$-score of a test with a score of 75 if the mean on this test is 80 with a standard deviation of 2.5?

# §Review

Describe in words what is meant by the sampling distribution of a median.

# §Review

Make sure to know all theorems, rules, and laws!