Lecture 9:
Chapter 9

C C Moxley

UAB Mathematics

22 October 15

§9.1 Testing Claims About Two Populations

In this chapter we will extend our inferential statistical methods to testing claims about **two** populations' parameters. We will test claims like the following.

§9.1 Testing Claims About Two Populations

In this chapter we will extend our inferential statistical methods to testing claims about **two** populations' parameters. We will test claims like the following.

- When people are given a $1 bill rather than four quarters, those who are given four quarters are less likely to spend the money.

§9.1 Testing Claims About Two Populations

In this chapter we will extend our inferential statistical methods to testing claims about **two** populations' parameters. We will test claims like the following.

- When people are given a \$1 bill rather than four quarters, those who are given four quarters are less likely to spend the money.
- A certain vaccine makes children less likely to contract the measles.

§9.1 Testing Claims About Two Populations

In this chapter we will extend our inferential statistical methods to testing claims about **two** populations' parameters. We will test claims like the following.

- When people are given a $1 bill rather than four quarters, those who are given four quarters are less likely to spend the money.
- A certain vaccine makes children less likely to contract the measles.
- Women's mean body temperatures are lower than men's.

§9.2 Claims About Two Proportions

The claim that people are less likely to spend money when given four quarters rather than a $1 bill is an example of a claim about the **proportion** of two populations. What are these two populations? The one given $1 bills and the one give four quarters. What would the null hypothesis and alternative hypothesis look like for this claim if we let p_1 be the proportion of people who spent money and had the $1 bill and p_2 be the proportion of people who spent money and had the four quarters?

§9.2 Claims About Two Proportions

The claim that people are less likely to spend money when given four quarters rather than a \$1 bill is an example of a claim about the **proportion** of two populations. What are these two populations? The one given \$1 bills and the one give four quarters. What would the null hypothesis and alternative hypothesis look like for this claim if we let p_1 be the proportion of people who spent money and had the \$1 bill and p_2 be the proportion of people who spent money and had the four quarters?

$$H_0 : p_1 = p_2$$

§9.2 Claims About Two Proportions

The claim that people are less likely to spend money when given four quarters rather than a $1 bill is an example of a claim about the **proportion** of two populations. What are these two populations? The one given $1 bills and the one give four quarters. What would the null hypothesis and alternative hypothesis look like for this claim if we let p_1 be the proportion of people who spent money and had the $1 bill and p_2 be the proportion of people who spent money and had the four quarters?

$$H_0 : p_1 = p_2$$
$$H_1 : p_1 > p_2$$

§9.2 Claims About Two Proportions

The claim that people are less likely to spend money when given four quarters rather than a $1 bill is an example of a claim about the **proportion** of two populations. What are these two populations? The one given $1 bills and the one give four quarters. What would the null hypothesis and alternative hypothesis look like for this claim if we let p_1 be the proportion of people who spent money and had the $1 bill and p_2 be the proportion of people who spent money and had the four quarters?

$$H_0 : p_1 - p_2 = 0$$
$$H_1 : p_1 - p_2 > 0$$

§9.2 Claims About Two Proportions

We can test this claim if we have samples from these two populations meeting the following criteria.

§9.2 Claims About Two Proportions

We can test this claim if we have samples from these two populations meeting the following criteria.

- The samples are independent, simple random samples.

§9.2 Claims About Two Proportions

We can test this claim if we have samples from these two populations meeting the following criteria.

- The samples are independent, simple random samples.
- There are at least five successes and five failures in each of the two samples.

§9.2 Claims About Two Proportions

We can test this claim if we have samples from these two populations meeting the following criteria.

- The samples are independent, simple random samples.
- There are at least five successes and five failures in each of the two samples.

We need the following to conduct this test.

§9.2 Claims About Two Proportions

We can test this claim if we have samples from these two populations meeting the following criteria.

- The samples are independent, simple random samples.
- There are at least five successes and five failures in each of the two samples.

We need the following to conduct this test.

Z-stat	$\hat{p}_1$ **and** $\hat{p}_2$	$\bar{p}$	E
$\dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$	$\dfrac{x_1}{n_1}$ and $\dfrac{x_2}{n_2}$	$\dfrac{x_1 + x_2}{n_1 + n_2}$	$z_{\frac{\alpha}{2}} \sqrt{\dfrac{\hat{p}_1 \hat{q}_1}{n_1} + \dfrac{\hat{p}_2 \hat{q}_2}{n_2}}$

In a group of 100 people given $1 bills, 28 spent money. In a group of 121 people given four quarters, 35 spent money. Test the claim that the proportion of people who spent money when given four quarters is higher than the proportion of people who spent money when given a dollar bill. Use significance $\alpha = 0.1$.

In a group of 100 people given $1 bills, 28 spent money. In a group of 121 people given four quarters, 35 spent money. Test the claim that the proportion of people who spent money when given four quarters is higher than the proportion of people who spent money when given a dollar bill. Use significance $\alpha = 0.1$.

$$\hat{p}_1 = \frac{28}{100} = 0.28,$$

In a group of 100 people given \$1 bills, 28 spent money. In a group of 121 people given four quarters, 35 spent money. Test the claim that the proportion of people who spent money when given four quarters is higher than the proportion of people who spent money when given a dollar bill. Use significance $\alpha = 0.1$.

$$\hat{p}_1 = \frac{28}{100} = 0.28, \ \hat{p}_2 = \frac{35}{121} = 0.28926$$

In a group of 100 people given \$1 bills, 28 spent money. In a group of 121 people given four quarters, 35 spent money. Test the claim that the proportion of people who spent money when given four quarters is higher than the proportion of people who spent money when given a dollar bill. Use significance $\alpha = 0.1$.

$$\hat{p}_1 = \frac{28}{100} = 0.28,\ \hat{p}_2 = \frac{35}{121} = 0.28926 \text{ and } \bar{p} = \frac{28+35}{100+121} = 0.28507.$$

§9.2 Claims About Two Proportions

In a group of 100 people given $1 bills, 28 spent money. In a group of 121 people given four quarters, 35 spent money. Test the claim that the proportion of people who spent money when given four quarters is higher than the proportion of people who spent money when given a dollar bill. Use significance $\alpha = 0.1$.

$$\hat{p}_1 = \frac{28}{100} = 0.28,\ \hat{p}_2 = \frac{35}{121} = 0.28926 \text{ and } \bar{p} = \frac{28+35}{100+121} = 0.28507.$$

Thus, we have

$$z = \frac{-0.00926}{\sqrt{\frac{(0.28507)(0.71493)}{100} + \frac{(0.28507)(0.71493)}{121}}} = -0.15182.$$

§9.2 Claims About Two Proportions

In a group of 100 people given \$1 bills, 28 spent money. In a group of 121 people given four quarters, 35 spent money. Test the claim that the proportion of people who spent more money when given four quarters is higher than the proportion of people who spent money when given a dollar bill. Use significance $\alpha = 0.1$.

Since $z = -0.15182$, we get that out P-value is $P(X \leq -0.15182) = 0.4396$.

§9.2 Claims About Two Proportions

In a group of 100 people given \$1 bills, 28 spent money. In a group of 121 people given four quarters, 35 spent money. Test the claim that the proportion of people who spent more money when given four quarters is higher than the proportion of people who spent money when given a dollar bill. Use significance $\alpha = 0.1$.

Since $z = -0.15182$, we get that out P-value is $P(X \leq -0.15182) = 0.4396$. Because the P-value is greater than α, we fail to reject our null hypothesis. This means that there is **not enough evidence to support the claim** that $p_1 < p_2$, i.e. our original claim (that it's more likely for someone to spend money when given four quarters than when given a dollar bill) is not supported by this data.

§9.2 Claims About Two Proportions

We can do this in StatCrunch as well! Let's try testing this same hypothesis by constructing a confidence interval for $p_1 - p_2$ in StatCrunch!

We can do this in StatCrunch as well! Let's try testing this same hypothesis by constructing a confidence interval for $p_1 - p_2$ in StatCrunch! The 90% confidence interval we get is $(-0.11, 0.09)$.

§9.2 Claims About Two Proportions

We can do this in StatCrunch as well! Let's try testing this same hypothesis by constructing a confidence interval for $p_1 - p_2$ in StatCrunch! The 90% confidence interval we get is $(-0.11, 0.09)$. Notice, this interval contains 0, which was our null hypothesis. Thus, we do not reject the null hypothesis, just as before.

§9.2 Claims About Two Proportions

We can do this in StatCrunch as well! Let's try testing this same hypothesis by constructing a confidence interval for $p_1 - p_2$ in StatCrunch! The 90% confidence interval we get is $(-0.11, 0.09)$. Notice, this interval contains 0, which was our null hypothesis. Thus, we do not reject the null hypothesis, just as before. Note: The significance of the hypothesis test corresponding to this 90% CI is not 10%. What is it?

§9.2 Claims About Two Proportions

Finally, we could test the hypothesis using the critical value method.

Finally, we could test the hypothesis using the critical value method. The Z-stat from above was $z = -0.15182$. Because the left-tailed critical value corresponding to $\alpha = 0.1$ is -1.28, we see that the Z-stat does not lie beyond the critical value, and so we do not reject the null hypothesis, just as before.

§9.3 Claims About Two Independent Sample Means

There are equations for the T-statistic, Z-statistic, degrees of freedom, etc., in the text. I am omitting them here - please look over them. I will emphasize the use of technology instead.

§9.3 Claims About Two Independent Sample Means

There are equations for the T-statistic, Z-statistic, degrees of freedom, etc., in the text. I am omitting them here - please look over them. I will emphasize the use of technology instead.

First, note the requirements for these types of hypothesis tests.

§9.3 Claims About Two Independent Sample Means

There are equations for the T-statistic, Z-statistic, degrees of freedom, etc., in the text. I am omitting them here - please look over them. I will emphasize the use of technology instead.

First, note the requirements for these types of hypothesis tests.

- The samples both come from a normally distributed population or are samples with size more than 30.
- Both samples are independent simple random samples.

§9.3 Claims About Two Independent Sample Means

If we do not know σ_1 or σ_2 and if we assume that they are different from one another, then we use a T test with **unpooled** sample variance.

§9.3 Claims About Two Independent Sample Means

If we do not know σ_1 or σ_2 and if we assume that they are different from one another, then we use a T test with **unpooled** sample variance.

If we know both σ_1 or σ_2, then we use a Z test.

§9.3 Claims About Two Independent Sample Means

If we do not know σ_1 or σ_2 and if we assume that they are different from one another, then we use a T test with **unpooled** sample variance.

If we know both σ_1 or σ_2, then we use a Z test.

If we do not know σ_1 or σ_2 and we assume that they are equal to each other, then we use a T test with **pooled** sample variance.

§9.3 Claims About Two Independent Sample Means

Researchers conducted a study to determine if olive oil was antibacterial. The first sample consisted of 12 cultures treated with olive oil. The mean number of bacteria in the olive oil cultures was 0.57 million, with a standard deviation of 0.54 million. The second sample consisted of 12 untreated cultures. The mean number of bacteria in the untreated cultures was 0.67 million with standard deviation of 0.89 million. Assume that the populations of bacteria in these cultures in normally distributed.

Researchers conducted a study to determine if olive oil was antibacterial. The first sample consisted of 12 cultures treated with olive oil. The mean number of bacteria in the olive oil cultures was 0.57 million, with a standard deviation of 0.54 million. The second sample consisted of 12 untreated cultures. The mean number of bacteria in the untreated cultures was 0.67 million with standard deviation of 0.89 million. Assume that the populations of bacteria in these cultures in normally distributed. Test the claim with $\alpha = 0.05$.

§9.3 Claims About Two Independent Sample Means

Researchers conducted a study to determine if olive oil was antibacterial. The first sample consisted of 12 cultures treated with olive oil. The mean number of bacteria in the olive oil cultures was 0.57 million, with a standard deviation of 0.54 million. The second sample consisted of 12 untreated cultures. The mean number of bacteria in the untreated cultures was 0.67 million with standard deviation of 0.89 million. Assume that the populations of bacteria in these cultures in normally distributed. Test the claim with $\alpha = 0.05$.

$$H_0 : \mu_1 - \mu_2 = 0$$

§9.3 Claims About Two Independent Sample Means

Researchers conducted a study to determine if olive oil was antibacterial. The first sample consisted of 12 cultures treated with olive oil. The mean number of bacteria in the olive oil cultures was 0.57 million, with a standard deviation of 0.54 million. The second sample consisted of 12 untreated cultures. The mean number of bacteria in the untreated cultures was 0.67 million with standard deviation of 0.89 million. Assume that the populations of bacteria in these cultures in normally distributed. Test the claim with $\alpha = 0.05$.

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_1 : \mu_1 < \mu_2$$

§9.3 Claims About Two Independent Sample Means

We have

	Sample 1	Sample 2
$\bar{x}$	0.57	0.67
s	0.54	0.89
n	12	12

And we use a T-stat with unpooled variance, getting $t = -0.333$ and $P = 0.3716$.

§9.3 Claims About Two Independent Sample Means

We have

	Sample 1	Sample 2
$\bar{x}$	0.57	0.67
s	0.54	0.89
n	12	12

And we use a T-stat with unpooled variance, getting $t = -0.333$ and $P = 0.3716$. Thus, we fail to reject the null hypothesis. There is not enough evidence to support the claim that the average number of bacteria in a culture treated with olive oil is less than the average number of bacteria in a culture left untreated.

A candy manufacturer makes blue candies and red candies. They are made on the same machinery which ensures that they vary in size by the same amount. But they are made of different material, so the manufacturer is unsure which is heavier. She suspects they have the same weight. She samples 301 blue candies and 205 red candies, discovering that they have a mean weight of 1.50 grams and 1.40 grams respectively and standard deviations of 0.049 and 0.051 grams respectively. Test this claim with significance $\alpha = 0.01$. Use a confidence interval.

§9.3 Claims About Two Independent Sample Means

A candy manufacturer makes blue candies and red candies. They are made on the same machinery which ensures that they vary in size by the same amount. But they are made of different material, so the manufacturer is unsure which is heavier. She suspects they have the same weight. She samples 301 blue candies and 205 red candies, discovering that they have a mean weight of 1.50 grams and 1.40 grams respectively and standard deviations of 0.049 and 0.051 grams respectively. Test this claim with significance $\alpha = 0.01$. Use a confidence interval.

$$H_0 : \mu_1 - \mu_2 = 0$$

A candy manufacturer makes blue candies and red candies. They are made on the same machinery which ensures that they vary in size by the same amount. But they are made of different material, so the manufacturer is unsure which is heavier. She suspects they have the same weight. She samples 301 blue candies and 205 red candies, discovering that they have a mean weight of 1.50 grams and 1.40 grams respectively and standard deviations of 0.049 and 0.051 grams respectively. Test this claim with significance $\alpha = 0.01$. Use a confidence interval.

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_0 : \mu_1 - \mu_2 \neq 0$$

§9.3 Claims About Two Independent Sample Means

We have

	Sample 1	Sample 2
$\bar{x}$	1.50	1.40
s	0.049	0.051
n	301	205

And we use a T-stat with pooled variance, getting the confidence interval (0.08,0.11).

§9.3 Claims About Two Independent Sample Means

We have

	Sample 1	Sample 2
$\bar{x}$	1.50	1.40
s	0.049	0.051
n	301	205

And we use a T-stat with pooled variance, getting the confidence interval (0.08,0.11). Thus, we reject the null hypothesis because the interval contains only positive values. There is evidence to support the claim that the mean weights of the blue and red candies are different.

We have

	Sample 1	Sample 2
$\bar{x}$	1.50	1.40
s	0.049	0.051
n	301	205

And we use a T-stat with pooled variance, getting the confidence interval (0.08,0.11). Thus, we reject the null hypothesis because the interval contains only positive values. There is evidence to support the claim that the mean weights of the blue and red candies are different. Which candy likely has the higher average weight?

§9.3 Claims About Two Independent Sample Means

We have

	Sample 1	Sample 2
$\bar{x}$	1.50	1.40
s	0.049	0.051
n	301	205

And we use a T-stat with pooled variance, getting the confidence interval (0.08,0.11). Thus, we reject the null hypothesis because the interval contains only positive values. There is evidence to support the claim that the mean weights of the blue and red candies are different. Which candy likely has the higher average weight? The blue candies (because the interval contained only positive values).

§9.3 Claims About Two Independent Sample Means

Make sure that, when doing a two-tailed test via a confidence interval, that you ensure an area of α is to the left and right of the interval!

§9.4 Claims About Two Dependent Samples

Two samples are dependent if they have a natural pairing! Which of the two pairs are independent? Dependent?

§9.4 Claims About Two Dependent Samples

Two samples are dependent if they have a natural pairing! Which of the two pairs are independent? Dependent?

- Fifteen voters in Ohio and fifteen voters in Indiana have their IQs tested.

§9.4 Claims About Two Dependent Samples

Two samples are dependent if they have a natural pairing! Which of the two pairs are independent? Dependent?

- Fifteen voters in Ohio and fifteen voters in Indiana have their IQs tested.
- Fifteen pairs of brothers and sisters have their IQs tests.

§9.4 Claims About Two Dependent Samples

Two samples are dependent if they have a natural pairing! Which of the two pairs are independent? Dependent?

- Fifteen voters in Ohio and fifteen voters in Indiana have their IQs tested.
- Fifteen pairs of brothers and sisters have their IQs tests.

The first are independent and the second are dependent.

§9.4 Claims About Two Dependent Samples

We need the following when conducting hypothesis tests with two dependent samples.

§9.4 Claims About Two Dependent Samples

We need the following when conducting hypothesis tests with two dependent samples.

- d, the difference between two pairs

§9.4 Claims About Two Dependent Samples

We need the following when conducting hypothesis tests with two dependent samples.

- d, the difference between two pairs
- μ_d, the mean of these differences for the population

§9.4 Claims About Two Dependent Samples

We need the following when conducting hypothesis tests with two dependent samples.

- d, the difference between two pairs
- μ_d, the mean of these differences for the population
- $\bar{d}$, the mean of these differences for the samples

§9.4 Claims About Two Dependent Samples

We need the following when conducting hypothesis tests with two dependent samples.

- d, the difference between two pairs
- μ_d, the mean of these differences for the population
- $\bar{d}$, the mean of these differences for the samples
- s_d, the standard deviation of these differences for the sample

§9.4 Claims About Two Dependent Samples

We need the following when conducting hypothesis tests with two dependent samples.

- d, the difference between two pairs
- μ_d, the mean of these differences for the population
- $\bar{d}$, the mean of these differences for the samples
- s_d, the standard deviation of these differences for the sample
- n, the number of pairs of sample data

§9.4 Claims About Two Dependent Samples

The following criteria must be met to test claims about the mean of two dependent samples.

§9.4 Claims About Two Dependent Samples

The following criteria must be met to test claims about the mean of two dependent samples.

- The sample data must be matched pairs.

§9.4 Claims About Two Dependent Samples

The following criteria must be met to test claims about the mean of two dependent samples.

- The sample data must be matched pairs.
- The samples must be simple random samples.

§9.4 Claims About Two Dependent Samples

The following criteria must be met to test claims about the mean of two dependent samples.

- The sample data must be matched pairs.
- The samples must be simple random samples.
- Either the differences must be normally distributed or the number of pairs must be larger than 30.

§9.4 Claims About Two Dependent Samples

We would like to test the claim that presidents tend to be taller than their main opponent, i.e. the taller candidate wins, with a significance level $\alpha = 0.05$. We have the following five pairs of data (whose differences are normally distributed).

Height P:	189	173	183	180	179
Height O:	170	185	175	180	178

We would like to test the claim that presidents tend to be taller than their main opponent, i.e. the taller candidate wins, with a significance level $\alpha = 0.05$. We have the following five pairs of data (whose differences are normally distributed).

Height P:	189	173	183	180	179
Height O:	170	185	175	180	178

We use StatCruch to calculate the differences and the test based on these differences.

§9.4 Claims About Two Dependent Samples

We would like to test the claim that presidents tend to be taller than their main opponent, i.e. the taller candidate wins, with a significance level $\alpha = 0.05$. We have the following five pairs of data (whose differences are normally distributed).

Height P:	189	173	183	180	179
Height O:	170	185	175	180	178

We use StatCruch to calculate the differences and the test based on these differences. We want to test the hypothesis below.

§9.4 Claims About Two Dependent Samples

We would like to test the claim that presidents tend to be taller than their main opponent, i.e. the taller candidate wins, with a significance level $\alpha = 0.05$. We have the following five pairs of data (whose differences are normally distributed).

Height P:	189	173	183	180	179
Height O:	170	185	175	180	178

We use StatCruch to calculate the differences and the test based on these differences. We want to test the hypothesis below.

$$H_0 : \mu_d = 0$$

$$H_1 : \mu_d > 0$$

§9.4 Claims About Two Dependent Samples

From StatCrunch, we get the differences.

Height P:	189	173	183	180	179
Height O:	170	185	175	180	178
Differences:	19	-12	8	0	1

From StatCrunch, we get the differences.

Height P:	189	173	183	180	179
Height O:	170	185	175	180	178
Differences:	19	-12	8	0	1

We also get a P-value of 0.2819, which tells us we should fail to reject our null hypothesis. This means our data does not support our original claim that presidents tend to be taller than their opponents.

§9 Mastery

Say you construct a 90% confidence interval for a the difference of two means, i.e. for $\mu_1 - \mu_2$ and that the confidence interval is $(-0.5, -0.1)$, what can be said about the relationship between the two means?

§9 Mastery

What's the difference between standard deviation and standard error?

The weights of 40 people are measured before and after a diet program, and the difference between their pre-program weight and post-program weight is recorded. We want to test the claim that the mean weight lost is at least 2.1lbs. We conduct a 5% significance hypothesis test to test this claim and get a test statistic of -2.034. Does this support our claim or not?

§9 Mastery

What's the difference between μ_d and $\bar{d}$? When is μ_d the parameter under consideration as opposed to $\mu_1 - \mu_2$?

§9 Mastery

To test the hypothesis that the weight of a first-born twin is less than the weight of the second-born twin, would you consider μ_d or $\mu_1 - \mu_2$ in your null and alternative hypotheses?

§9 Mastery

When conducting a critical value hypothesis test for a claim about μ_d, what gets compared to the critical value?

- The Z-test statistic.
- The T-test statistic.
- The value set equal to μ_d in the null hypothesis.
- Any value that satisfies the inequality in the alternative hypothesis.
- The P-value.

§9 Mastery

Why must we require that there are at least five successes and five failures in each sample when conducting a hypothesis test concerning two proportions? What distributions are we actually comparing in these types of hypothesis tests?

§9 Mastery

You want to determine if the proportion of peas with adequate iron levels grown in water is less than the proportion of peas with adequate iron levels grown in soil. You sample 500 peas grown in water and 470 peas grown in soil. 45% of peas grown in water contain adequate amounts of iron and 45.2% of peas grown in soil contain adequate amounts of iron. Does this support the claim you wish to test or not?

Whether the average heights of longleaf pines and the average heights of loblolly pines are the same. You know that loblolly and longleaf pines vary in height by different amounts. A sample of 35 longleafs gives an average height of 75ft with standard deviation of 18 inches. A sample of 32 loblollies gives an average height of 75.1 feet with standard deviation of 11 inches. Do these samples support the claim that the average height of these pines is the same?