Lecture 12:
Chapter 12

C C Moxley

UAB Mathematics

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§12.1 ANOVA (Analysis of Variance) Tests

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§12.1 ANOVA (Analysis of Variance) Tests

In Chapter 9, we tested to see if two population means were equal. In this chapter, we will test to see if three or more population samples are the same using a **one-way** ANOVA test. We call it "one-way" because we separate our populations into groups based on one characteristic. This is often called an "analysis of variance" but this refers to the method of testing, not the thing which we are testing - which is means not variances.

§12.2 One-Way ANOVA Test

Definition (One-Way ANOVA Test)

A one-way ANOVA test is a method of testing the equality of three or more population means by analyzing sample variances. One-way analysis of variance is used with data categorized with one **factor**, so there is one characteristic used to separate the sample data into the different categories.

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- Populations must be approximately normally distributed.
- They must have the same variance, roughly. (Or at least the sample sizes must be the same for each category.)
- The samples must be simple random and quantitative.
- The samples must be independent - not matched or paired.
- The different samples are from populations that are categorized in only one way.

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H_1 : At least one of the population means differs from the others.

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H_1 : At least one of the population means differs from the others.

You can find "by hand" computations for a one-way ANOVA test, see page 605. The test is a right-tailed F test.

§12.2 Example

The weights (in kg) of oak trees is given below for trees planted in the same plot but with different growth-enhancing methods applied. Use a 0.05 significance level to test the claim that the four treatment categories yield oak trees with the same mean weight. Does there appear to be a best method for enhancing growth of trees in this soil?

Control	Fert	Irr	Fert & Irr
0.24	0.92	0.96	1.07
1.69	0.07	1.43	1.63
1.23	0.56	1.26	1.39
0.99	1.74	1.57	0.49
1.80	1.13	0.72	0.95

§12.2 Example

Control	Fert	Irr	Fert & Irr
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We use StatCrunch to conduct the test.

§12.2 Example

What's our null and alternative hypotheses? Based on the output, do we reject or accept the null hypothesis? What does this mean? (Use $\alpha = 0.05$.)

Options

Analysis of Variance results:
Data stored in separate columns.

Column statistics

Column	n	Mean	Std. Dev.	Std. Error
Control	5	1.19	0.62573956	0.27983924
Fert	5	0.884	0.624924	0.27947451
Irr	5	1.188	0.34665545	0.15502903
F & I	5	1.106	0.43598165	0.19497692

ANOVA table

Source	DF	SS	MS	F-Stat	P-value
Columns	3	0.3114	0.1038	0.38010491	0.7687
Error	16	4.36932	0.2730825		
Total	19	4.68072			

§12.2 Example

You can also perform the previous test using a critical value test. The F critical value is computed using the significance, the F calculator, and the "degrees of freedom of the numerator" and the "degrees of freedom of the denominator" which are, respectively, one fewer than the number of columns and the difference between the total number of data points and the number of columns.

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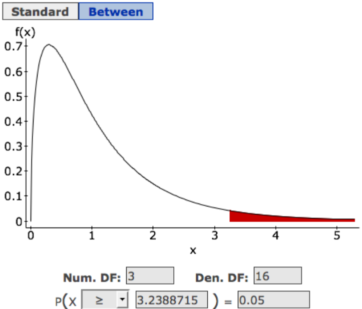

§12.2 Inference for One-Way ANOVA

An **analysis of variance (ANOVA)** test compares the means of multiple populations/subpopulations.

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$$\text{DATA} = \text{FIT} + \text{RESIDUAL}.$$

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$$\text{DATA} = \text{FIT} + \text{RESIDUAL}.$$

This model arises from the assumption that all the means are equal (our hull hypothesis). And thus, the sample means should follow

$$x_{ij} = \mu_i + \epsilon_{ij},$$

where $i = 1, \ldots, I$ and $j = 1, \ldots, n_i$ and ϵ_{ij} all have $N(0, \sigma)$ distribution.

§12.2 Inference for One-Way ANOVA

The estimate for σ is s_p which is given by

$$\sqrt{\frac{(n_1 - 1)s_1^2 + \cdots + (n_i - 1)s_i^2}{(n_1 - 1) + \cdots + (n_i - 1)}}.$$

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We always pool the standard deviation for a one-way ANOVA.

§12.2 Inference for One-Way ANOVA

Definition (hypothesis for one-way ANOVA)

The **null and alternative hypothesis** for a one-way ANOVA are

$$H_0 : \mu_1 = \cdots = \mu_i$$

$$H_a : \text{ not all of the } \mu_i \text{ are equal.}$$

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§12.2 Inference for One-Way ANOVA

Definition (sum of squares, degrees of freedom, and mean squares)

Sum of squares represent variation in the data. They are calculated by summing square deviations. There are three sources of variation in a one-way ANOVA.

$$\text{SST} = \text{SSG} + \text{SSE}$$

The **degrees of freedom** are associated with each sum of squares.

$$\text{DFT} = \text{DFG} + \text{DFE}$$

The **mean squares** are $\frac{\text{sum of squares}}{\text{degrees of freedom}}$.

Use the table below to calculate the values discussed in the previous slide.

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Source	DF	SS	MS	F
Groups	$I - 1$	$\sum_{\text{groups}} n_i(\bar{x}_i - \bar{x})^2$	$\frac{\text{SSG}}{\text{DFG}}$	MSG/MSE
Error	$N - I$	$\sum_{\text{groups}} (n_i - 1)s_i^2$	$\frac{\text{SSE}}{\text{DFE}}$	
Total	$N - 1$	$\sum_{\text{observations}} (x_{ij} - \bar{x})^2$		

§12.2 Inference for One-Way ANOVA

Definition (one-way ANOVA F test)

To test the null hypothesis that three or more population means are the same versus the alternative that at least one of them if different from the others, use the F statistic

$$F = \frac{\text{MSG}}{\text{MSE}}.$$

The P-value is the probability that a random variable having the $F(I-1, N-I)$ distribution is greater than or equal to the calculated value of the F statistic.

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This is a right-tailed F test.

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- The samples must be independent.
- There must be only a single factor separating populations.

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- Each population should be roughly normal.
- The samples must be independent.
- There must be only a single factor separating populations.
- The samples must be SRSs.

Example (mean credit score)

We want to see if the mean credit scores are the same across three age groups. We collect the following data from three SRSs.

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18-25	25-50	over 50
650	725	700
625	670	660
645	770	750
600	590	700
590	700	760
500		725
595		

Example (mean credit score)

We want to see if the mean credit scores are the same across three age groups. We collect the following data from three SRSs.

	18-25	25-50	over 50
$\bar{x}$	600.71429	691	715.83333
s	50.450353	67.305275	36.934627
n	7	5	6

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Thus,

$$s_p = \sqrt{\frac{6(50.450353)^2 + 4(67.305275)^2 + 5(36.934627)^2}{6+4+5}} = 51.777.$$

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$\bar{x}$	600.71429	691	715.83333
s	50.450353	67.305275	36.934627
n	7	5	6

Also, $\bar{x}_{\text{total}} = 664.16667$, so we have

$$\text{SSG} = 7(600.71429 - 664.16667)^2 + 5(691 - 664.16667)^2 +$$

$$6(715.83333 - 664.1667)^2 = 47800.214.$$

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We want to see if the mean credit scores are the same across three age groups. We collect the following data from three SRSs.

	18-25	25-50	over 50
$\bar{x}$	600.71429	691	715.83333
s	50.450353	67.305275	36.934627
n	7	5	6

Also, we have

$$\text{SSE} = 6(50.450353)^2 + 4(67.305275)^2 + 5(36.934627)^2 = 40212.262.$$

Example (mean credit score)

We want to see if the mean credit scores are the same across three age groups. We collect the following data from three SRSs.

	18-25	25-50	over 50
$\bar{x}$	600.71429	691	715.83333
s	50.450353	67.305275	36.934627
n	7	5	6

Thus, we must have $\text{SST} = 40212.262 + 47800.214 = 88012.5$.

§12.2 Inference for One-Way ANOVA

Example (mean credit score)

Let's fill in our table:

Source	DF	SS	MS	F
Groups	$I-1$	$\sum_{\text{groups}} n_i(\bar{x}_i - \bar{x})^2$	$\frac{\text{SSG}}{\text{DFG}}$	MSG/MSE
Error	$N-I$	$\sum_{\text{groups}} (n_i - 1)s_i^2$	$\frac{\text{SSE}}{\text{DFE}}$	
Total	$N-1$	$\sum_{\text{observations}} (x_{ij} - \bar{x})^2$		

Example (mean credit score)

Let's fill in our table:

Source	DF	SS	MS	F
Groups	2	47800.214	$\frac{SSG}{DFG}$	MSG/MSE
Error	$N - I$	$\sum_{\text{groups}} (n_i - 1)s_i^2$	$\frac{SSE}{DFE}$	
Total	$N - 1$	$\sum_{\text{observations}} (x_{ij} - \bar{x})^2$		

Example (mean credit score)

Let's fill in our table:

Source	DF	SS	MS	F
Groups	2	47800.214	$\frac{SSG}{DFG}$	MSG/MSE
Error	15	40212.262	$\frac{SSE}{DFE}$	
Total	$N-1$	$\sum_{\text{observations}} (x_{ij}-\bar{x})^2$		

Example (mean credit score)

Let's fill in our table:

Source	DF	SS	MS	F
Groups	2	47800.214	$\frac{SSG}{DFG}$	MSG/MSE
Error	15	40212.262	$\frac{SSE}{DFE}$	
Total	17	88012.5		

Example (mean credit score)

Let's fill in our table:

Source	DF	SS	MS	F
Groups	2	47800.214	23900.107	8.915
Error	15	40212.262	2680.817	
Total	17	88012.5		

Example (mean credit score)

Let's fill in our table:

Source	DF	SS	MS	F
Groups	2	47800.214	23900.107	8.915
Error	15	40212.262	2680.817	
Total	17	88012.5		

Thus, the P-value for this test is $P(F > 8.915) = 0.003$.

Example (mean credit score)

Let's fill in our table:

Source	DF	SS	MS	F
Groups	2	47800.214	23900.107	8.915
Error	15	40212.262	2680.817	
Total	17	88012.5		

Thus, the P-value for this test is $P(F > 8.915) = 0.003$. So if we conducted the one-way ANOVA at a 5% significance level, we would reject the null hypothesis. We would not support the claim that the mean credit scores are the same across all three of these age groups.

§12.2 Inference for One-Way ANOVA

Note: The full one-way ANOVA test can be conducted in StatCrunch! You should make sure that you know how the one-way ANOVA table could be filled in given just a few SS and DF values, though!

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Note: The full one-way ANOVA test can be conducted in StatCrunch! You should make sure that you know how the one-way ANOVA table could be filled in given just a few SS and DF values, though! Let's do one more test using Statcrunch.

§12.2 Inference for One-Way ANOVA

Example (labor costs)

A company would like to see if its labor costs are the same over all four seasons. It takes a SRS of labor costs in each season over several years and gets the following data.

Spring	Summer	Fall	Winter
300000	295000	259000	260000
450000	430000	460000	475000
375000	380000	385000	380000
257000	277000	259000	300000
280000	285000	275000	290000
300000	299000	301000	298000
440000	445000	440500	444000

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Example (labor costs)

A company would like to see if its labor costs are the same over all four seasons. It takes a SRS of labor costs in each season over several years and gets the following data. We get the following table.

ANOVA table

Source	DF	SS	MS	F-Stat	P-value
Columns	3	3.3774107e8	1.1258036e8	0.017332332	0.9968
Error	24	1.558895e11	6.4953958e9		
Total	27	1.5622724e11			

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ANOVA table

Source	DF	SS	MS	F-Stat	P-value
Columns	3	3.3774107e8	1.1258036e8	0.017332332	0.9968
Error	24	1.558895e11	6.4953958e9		
Total	27	1.5622724e11			

Thus, we fail to reject the null hypothesis and support the claim that the labor costs are the same across all four seasons.