Lecture 3:
Chapter 3

C C Moxley

UAB Mathematics

26 January 16

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- mean
- median
- mode
- midrange
- weighted mean

§3.2 Mean

Definition (Mean)

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Example (Mean)

A sample of 10 fun sized candy bags showed that each bag had the following number of candies: 12, 12, 12, 13, 14, 14, 15, 15, 16, 16. What's the average number of candies per bag for this sample?

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$$m = \frac{12+12+12+13+14+14+15+15+16+16}{10} = 13.9.$$

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Example (Median)

A sample of 10 fun sized candy bags showed that each bag had the following number of candies: 12, 12, 12, 13, 14, 14, 15, 15, 16, 16. What's the median number of candies in a bag? These are already ordered! Because there is an even number, the median is the average of the 5th and 6th data points, i.e. it is the average of 14 and 14, which is 14.

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Example (Mode)

A sample of 10 fun sized candy bags showed that each bag had the following number of candies: 12, 12, 12, 13, 14, 14, 15, 15, 16, 16. What's the mode of this data set? The mode is 12.

§3.2 Midrange and Weighted Average

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- The midrange is the point directly between the lowest and highest data points, i.e. it is the average of the max and the min of a data set.
- A weighted average places greater or smaller significance on certain data points. (An example would be GPA calculation.)

§3.3 Measures of Variation

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1, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 9.

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1, 1, 2, 2, 3, 3, 4, 5, 5, 5, 5, 6, 7, 7, 8, 8, 9, 9.

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Example (Range)

What is the range of the data set 1, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 9? It's $9 - 1 = 8$.

§3.3 Standard Deviation

Definition (Standard Deviation)

The **standard deviation** of a set of data points is given by

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{n\sum_{i=1}^{n}x_i^2 - \left(\sum_{i=1}^{n}x_i\right)^2}{n(n-1)}}$$

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The standard deviation measures how far the data points vary from the **mean**. When is it zero? Is it ever negative?

Example (Calculate Standard Deviation)

Use 1, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 9.

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$$s = \sqrt{\frac{18(492) - 90^2}{18(17)}} \approx 1.57$$

Example (Calculate Standard Deviation)

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Use 1, 1, 2, 2, 3, 3, 4, 5, 5, 5, 5, 6, 7, 7, 8, 8, 9, 9. Well, $\sum_{i=1}^{n} x_i^2 = 568$ and $\bar{x} = 5$, so we have

$$s = \sqrt{\frac{18(568) - 90^2}{18(17)}} \approx 2.63$$

§3.3. Standard Deviation

How is the standard deviation useful?

- *When means are similar*, you can use the standard deviation to see differences in variation in samples.
- The standard deviation is less sentitive than range for measuring variation.
- As a general rule of thumb, you should expect to see about 95% of all data points falling within 2 standard deviations of the mean.

§3.3. Population Standard Deviation

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What are the differences between population standard deviation and standard deviation?

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- Variance is more sensitive to outliers than is standard deviation.
- Variance carries the square of the units of the data it describes.
- Variance is an unbiased estimator while standard deviation is a biased estimator (in the sense described later).
- Variance is always non-negative.

§3.3. Bias

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§3.3. Chebyshev Theorem and Bell-Shaped Distributions

Theorem (Chebyshev's Theorem)

The proportion of any set of data lying within K standard deviations of the mean is always at least $1 - 1/K^2$, where $K > 1$.

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- $\approx 68\%$ *of all values lie within s of m,*
- $\approx 95\%$ *of all values lie within $2s$ of m, and*
- $\approx 99.7\%$ *of all values lie within $3s$ of m.*

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$$CV = \frac{s}{\bar{x}} \cdot 100\% \qquad CV = \frac{\sigma}{\mu} \cdot 100\%$$

Example (Comparing apples and oranges)

Apples weigh an average of 7 ounces with a standard deviation of 2.1 ounces. Oranges have an average volume of 190 mL with a standard deviation of 2.1 mL.

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Example (Comparing apples and oranges)

Apples weigh an average of 7 ounces with a standard deviation of 2.1 ounces. Oranges have an average volume of 190 mL with a standard deviation of 2.1 mL. Because $CV_A \approx 30\%$ whereas $CV_O \approx 1.1\%$, we know that apples vary in weight much more than oranges vary in volume.

§3.4 Measures of Relative Standing

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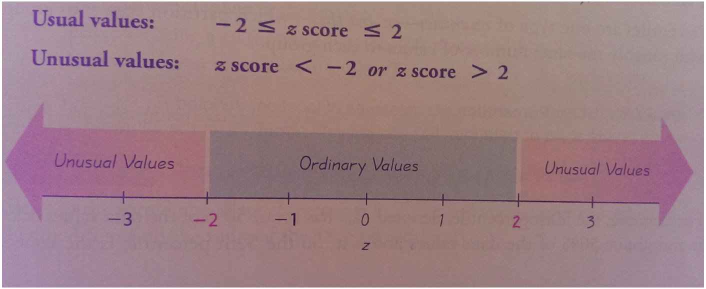
The z-score for a datum is a how many standard deviations it is above or below the mean.

$$z = \frac{x - \bar{x}}{s}, \qquad z = \frac{x - \mu}{\sigma}$$

‘Typical’ values have a z-score of absolute value less than or equal to 2.



§3.4 Percentiles/Quantiles/Quartiles

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$$\text{percentile value of } x = \frac{\text{number of values less than } x}{\text{total number of values}} \cdot 100$$

First, second, and third quartiles are P_{25}, P_{50}, and P_{75}, respectively.

Briefly describe interquartile range, semi-interquartile range, midquartile, and 10-90 percentile range.

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