Lecture 4:
Chapter 4

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UAB Mathematics

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§4.2 Basic Concepts of Probability

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three tests	PPP or FFF	PFP	$\{\text{PPP}, \text{PPF}, \ldots, \text{FFF}\}$

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- **Subjective probabilities:** Estimate $P(A)$ by using knowledge of relevant circumstances.

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$$\frac{205}{1010} \approx 20.3\%$$

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Use the subjective probability method: Only 1 in 100000 people in the US own both a cane and tophat. The probability is very small, maybe 0.00001.

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Use the classical approach. There are 52^2 possible outcomes, of which 4^2 are drawing a king from the first deck and a king from the second deck. So, the probability is $\frac{16}{2704} \approx 0.59\%$.

§4.2 Definitions

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Discuss the rare event rule and how it is used to investigate hypotheses.

§4.3 Addition Rule

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What is the chance that a randomly selected member of this sample had either contacts or glasses?

$$P(\text{contacts or glasses}) = \frac{30}{50} + \frac{35}{50} - \frac{23}{50} = \frac{42}{50}.$$

§4.3 Complementary Events Rule

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Example (Find $P(\overline{A \text{ or } B})$.)

$P(\overline{A \text{ or } B}) = 1 - P(A) - P(B) + P(A \text{ and } B)$.

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Discuss $P(A \text{ and } B)$ and $P(B|A)$.

The probability that two events occur is equal to the probability that the first occurs times the probability that the second occurs **if these events are independent**!

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What is the probability that two randomly selected grades are both Bs?

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Example (What are the chances that 26 randomly chosen people have all different birthdays?)

$$\frac{(365)(364)(363)\ldots(341)(340)}{(365)(365)(365)\ldots(365)(365)} = \frac{(365)(364)(363)\ldots(341)(340)}{365^{26}} \approx 40.18\%$$

§4.4 Multiplication Rule: Redundancy

The multiplication rule for independent events helps illustrate why some important industrial components have redundancy: If an oil pipeline has five different oil pressure measuring tools to ensure that the pipeline is not leaking oil and if each of these tools has a fail rate of 5%, then what is the probability that oil is leaking without being detected?

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$$0.05^5 = 0.0000003125.$$

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- First, compute $P(\bar{A})$.
- Then subtract it from 1 because $P(A) = 1 - P(\bar{A})$.

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Now, if we wanted to compute the probability that at least one birthday is shared amongst 26 people (which we will call event A), we can calculate

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$$1 - 0.4018 = 0.5982.$$

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$$P(A|B) = \frac{55}{505} = 0.1089.$$

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$$P(A|B) = \frac{450}{453} = 0.9934.$$

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Warning: $P(A|B) \neq P(B|A)$!

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5. $\frac{n!}{(n-r)!r!} =$ number of different combinations of r items chosen without replacement from n different items.

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Why might we not want to conduct a census? What can we do instead of a census? What are the pros and cons of a census versus our other option?

What is the difference between a simple random sample and a random sample?

A magazine asked its readers to fill out and mail in a survey on the last page of its latest issue. What are the pitfalls of this survey?

Are pictographs useful? Why might you try to use one?

Are pictographs useful? Why might you try to use one? Why might you break an axis in a bar graph?

What are the original values of the data resulting in the z-scores 0.5, 0.25, and -0.75 if the data came from a population with mean 0 and standard deviation 4?

§Test 1 Review

Define statistical significance. Can something be statistically significant without being practically significant?

What are the units of standard deviation if the original data has units square inches? What are the units of variance in this case?

§Test 1 Review

Pair together the similar measurements: third quartile, first quartile, second quartile, median, fiftieth percentile, twenty-fifth percentile, seventy-fifth percentile.

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Can you discard outliers to clean up data sets?