

Lecture 7: Chapter 7

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UAB Mathematics

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§7.1 Inferential Statistics

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Definition (Inferential Statistics)

The use of sample data to make statements about the statistical/probabilistic characteristics of a population.

§7.2 Estimating a Population Proportion

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E.g. 35% (or 0.35) is a point estimate.

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E.g. 95% (or 0.95) is a confidence level. In this case, $\alpha = 0.05$. Often, you are asked to create the confidence interval given a certain desired confidence level. In this chapter, we use confidence intervals for informal hypothesis testing.

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Definition (Critical Value)

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Different distributions must have critical values calculated differently!

§7.2 Estimating a Population Proportion

Definition (Margin of Error)

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You can also use this information to calculate how big a sample size needs to be to yield a certain level of confidence with a certain error size. (Always round up when doing this.)

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which implies that $n \approx 3269.78$, so we need a sample size of at least 3270.

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- There are at least five successes and five failures.

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$$n - 1,$$

where n is the sample size.

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50 body temperatures of cats were taken and their average was $101.5^{\circ}F$. It's known that cats have body temperatures which have a standard deviation of $0.7^{\circ}F$. Find a 95% confidence interval for the mean body temperature of cats.

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which means our sample size must be at least 133 cats.

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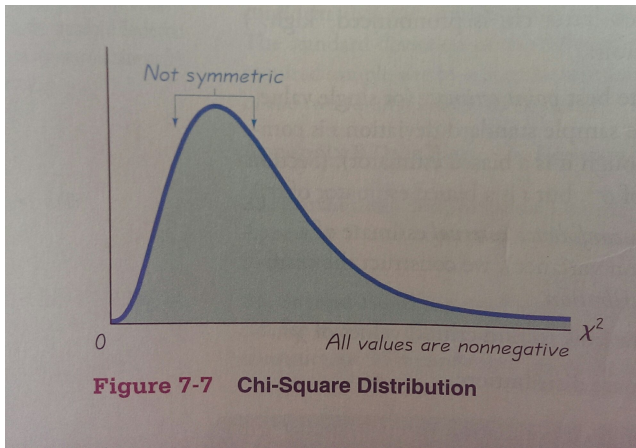
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§7.4 The χ^2 Distribution



§7.4 Estimating Population Standard Deviation or Variance

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$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}},$$

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and for the variance, we have

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- The population **must be normally distributed, even if the sample is large!**

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The time a student spends on a section of the SAT is given in minutes: 25, 21, 21, 18, 13, 13, 14, 16, 12, 22, 25, 25, 30. These times are known to be normally distributed. Calculate a 90% confidence interval for the standard deviation of these times.

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Therefore,

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}} \implies$$
$$4.35 = \sqrt{\frac{(12)5.752^2}{21.0261}} < \sigma < \sqrt{\frac{(12)5.752^2}{5.2260}} = 8.72.$$

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This means that if we took a large number of simple random samples of size 13 and calculated their standard deviation, then 90% of the time, your calculated standard deviation would fall into the interval $(4.35, 8.68)$.

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So we have that the 90% confidence interval is (4.35,8.68). What does this actually mean?

This means that if we took a large number of simple random samples of size 13 and calculated their standard deviation, then 90% of the time, your calculated standard deviation would fall into the interval (4.35,8.68). It does not mean that there is a 90% chance that σ falls within the interval.

§7.4 Precise Language

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What does it mean to have a 95% CI for population mean? It is precisely an interval which, when a random selection is made from the sampling distribution of the mean, will contain the given selection 95% of the time. We'll talk about how this relates to hypothesis testing in the next chapter, and it will be much easier to speak about what a 95% CI (or confidence level) means in the context of hypothesis testing.

In a probability histogram for the number of female girls in a family of 5, how would you determine the probability of having two or three females?

In some population, the mean is 25 and the standard deviation is 6. What is the probability that, in a simple random sample of 36, the sample mean would be between 24 and 26?

20% of students at UAB live on campus. Approximate the probability of choosing 10 students at random from the student population and having all of them live on campus.

What is the probability of having exactly one student who does not live on campus under the same circumstances as in the previous question?

What is the z-score of a test with a score of 75 if the mean on this test is 80 with a standard deviation of 2.5?

Describe in words what is meant by the sampling distribution of a median.

§Review

In a bag, you have 10 coins: 5 quarters, 1 dime, 1 nickel, and 3 pennies. If you pay 15 cents, you can draw one coin from the bag at random. Is this game a good game to pay many times?

The Central Limit Theorem says that when a sample is large the distribution of a sample mean is always what?

What's the probability of selecting two kings from a deck of cards with and without replacement? When does it make sense to compute sampling without replacement as if we are sampling with replacement?

How many ways are there of getting two heads when you're flipping a coin 5 times?

Make sure to know all theorems, rules, and laws!