Lecture 8:
Chapter 8

C C Moxley

UAB Mathematics

1 March 16

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- A sample of 103 human body temperatures can be used to test whether or not the mean body temperature for humans is 98.6°F.

§8.2 Basic Hypothesis Testing

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Example

From the previous, testing that the average weight of tennis balls manufactured by Wilson is less that 100 grams would be equivalent to testing the statement

$$\mu < 100,$$

where μ is the average weight of Wilson tennis balls.

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- The **null** hypothesis H_0 should always be that a population parameter is equal to some value.
- The **alternative** hypothesis H_1 should either be that the same parameter is not equal to, less than, or greater than the value above.

§8.2 Example

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- Testing using a critical value. This method is similar to constructing a confidence interval, except we may have one sided intervals now.
- Testing using a P-value, which describes the area lying beyond a test statistic in a one- or two-sided manner.

§8.2 Test Statistic

Definition (Test Statistic)

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Proportion p	Mean μ with σ	Mean μ w/o σ	Std. Dev. σ
$z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}$	$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$	$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$	$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$

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The significance of a P-test is called α. We reject the null hypothesis if $p \leq \alpha$ and fail to reject it if $p > \alpha$.

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- If the alternative hypothesis is a two-tailed ("not equal to") statement, then the P-value is the area outside of the interval $(-\omega, \omega)$ (if ω is positive) or $(\omega, -\omega)$ (if ω is negative). In the case of a χ^2 test, we look at the small tail and double the area in that tail for the P-value.

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Definition (Type II Error)

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This would be like concluding that a population proportion was 50% when it actually was not.

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The confidence of a test is $1-\alpha$ (the probability of failing to reject a true null hypothesis), and the power of a test is $1-\beta$ (the probability of rejecting a false null hypothesis).

§8.3 Example (Proportion)

A survey showed that, among the 514 human resource managers polled, 93% said that the appearance of a job applicant was important. Use a 10% significance level to test the claim that at least 95% of human resources managers think that a job applicant's appearance is important.

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And because $P(Z < -2.085) = 0.0186$, we get that we must reject the null hypothesis. In this case, this means that the data does **not** support our original claim!

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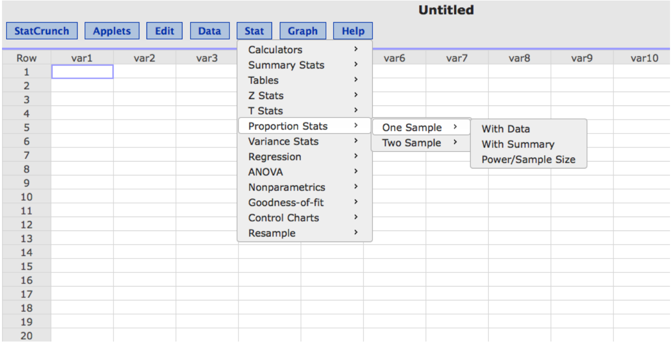

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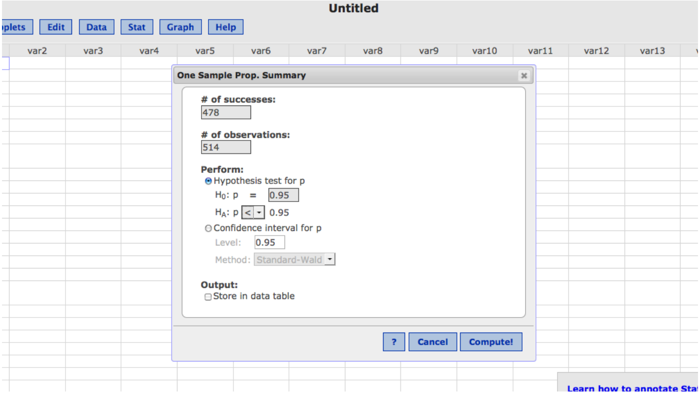

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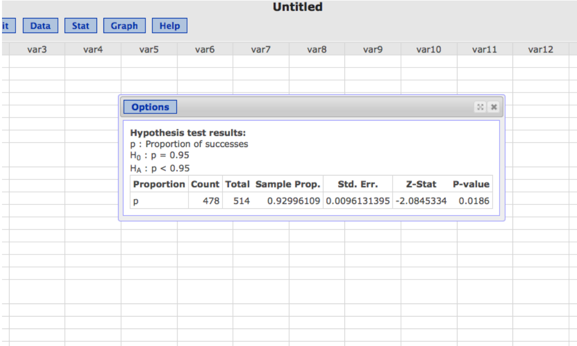

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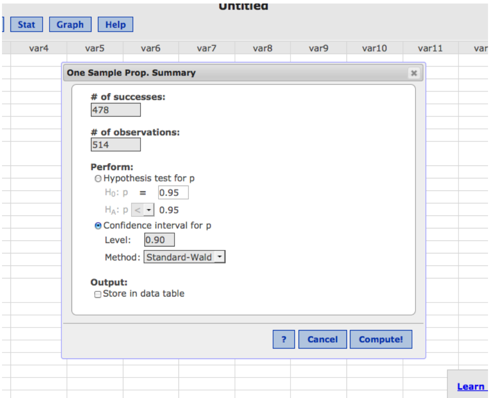

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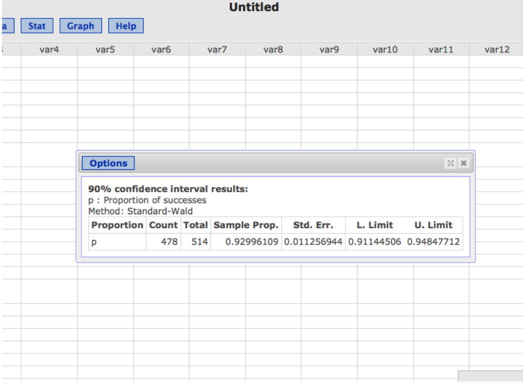

§8.3 Importance of Using an Appropriate Test

Notice the Confidence Interval Method resulting in failing to reject the null hypothesis because it is inherently **two-tailed**. **You must change the significance/confidence level when testing and one-tailed claim using a confidence interval!**

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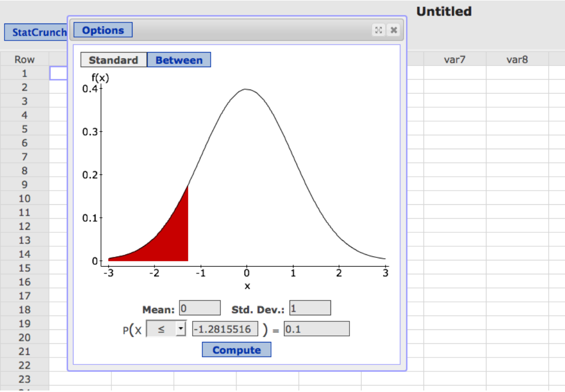

§8.4 Example (Mean)

Use a 0.05 significance level to test the claim that the data below comes from a normally distributed population with mean less than 1.

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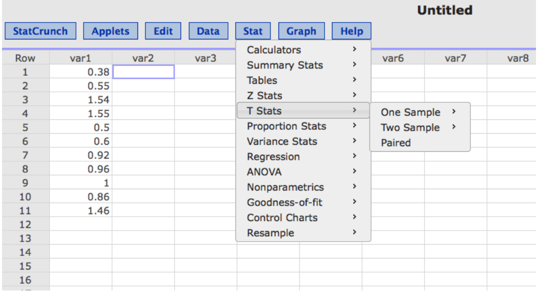

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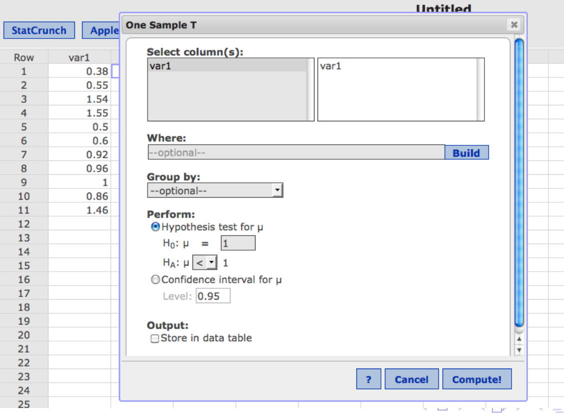

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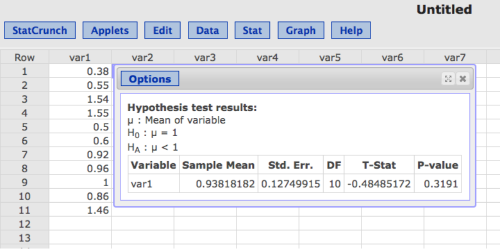

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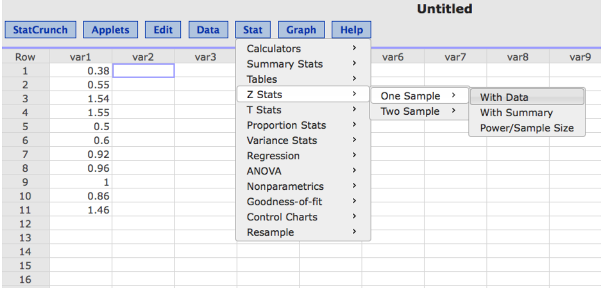

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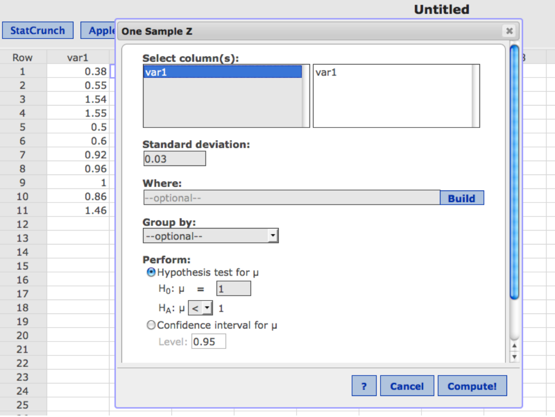

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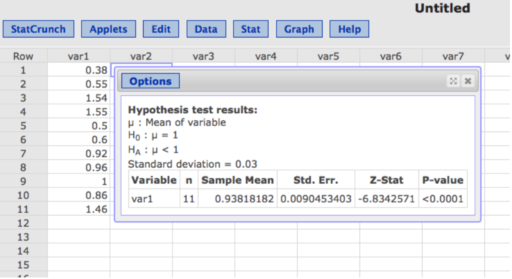

§8.4 Example (Standard Deviation)

Use a 0.01 significance level to test the claim that the data below comes from a normally distributed population with standard deviation equal to 2.6.

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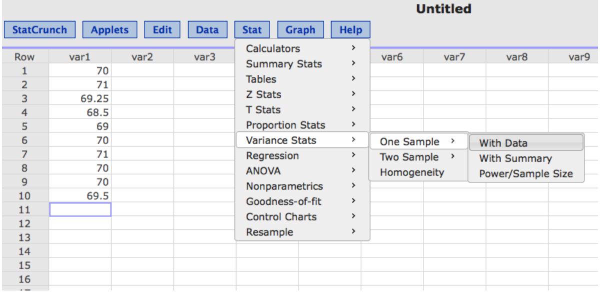

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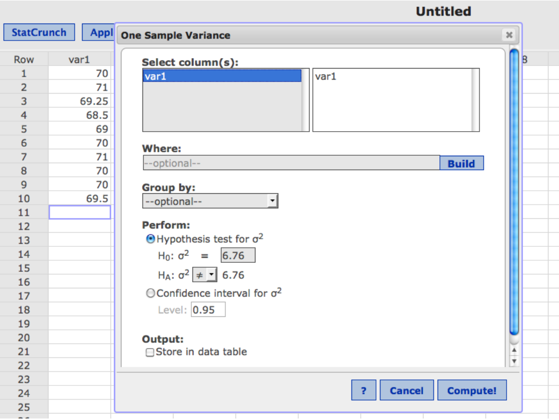

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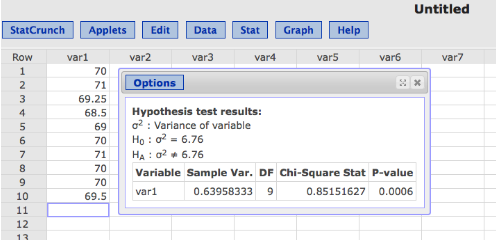

§8 Mastery

What would be a Type I error if the claim was that the proportion of people who write with their left hand is equal to 0.1? What would be a Type 2 error?

§8 Mastery

The claim is that women have heights with $\sigma = 5$cm. After the test is conducted, it's found that the P-value was 0.0055. If we're conducting a test with 99% confidence, do we support this claim or not?

We want to test the claim that at least 98% of Cheez-Its have at least 1.5mg of salt on them. In a sample of 120 Cheez-It crackers, we find that 118 have at least 1.5mg of salt on them. Do we support our claim or not?

§8 Mastery

The brain volumes of cows are given below. We want to test the claim that the population of cow brain volumes has mean equal to 1100 square centimeters. Assume brain volumes of cows are normally distributed.

$$963, 1027, 1272, 1079, 1070, 1173, 1067, 1347, 1100, 1204$$

The claim is that for nicotine amounts in a certain brand of cigarettes, $\mu > 20.0$mg. A test of 200 cigarettes showed the mean to be 20.1mg. The standard deviation for nicotine in this brand of cigarettes is 0.9mg. Can we support our claim with a 10% significance?

§8 Mastery

A simple random sample of 40 men results in a standard deviation of 10.3 heartbeats per minute. Men's heartbeats are normally distributed. We wish to test the claim that men's heartbeats have a standard deviation of 10 heartbeats per minute with a significance of 0.05.