Lecture 11:
Chapter 8

C C Moxley

UAB Mathematics

6 July 15

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- A paper claims that most American consumers know that the Kindle is a e-book reader.
- A sample of 103 human body temperatures can be used to test whether or not the mean body temperature for humans is 98.6°F.

§8.2 Basic Hypothesis Testing

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Example

From the previous, testing that the average weight of tennis balls manufactured by Wilson is less that 100 grams would be equivalent to testing the statement

$$\mu < 100,$$

where μ is the average weight of Wilson tennis balls.

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First we must form a hypothesis. Use these general rules:

- The **null** hypothesis H_0 should always be that a population parameter is equal to some value.
- The **alternative** hypothesis H_1 should either be that the same parameter is not equal to, less than, or greater than the value above.

§8.2 Example

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- Testing using a critical value. This method is essentially the same as constructing a confidence interval, except we may have one sided intervals now.
- Testing using a P-value, which describes the area lying beyond a test statistic in a one- or two-sided manner.

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Proportion p	Mean μ with σ	Mean μ w/o σ	Std. Dev. σ
$z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}$	$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$	$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$	$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$

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The significance of a P-test is called α. We reject the null hypothesis if $p \leq \alpha$ and fail to reject it if $p > \alpha$.

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- If the alternative hypothesis is a right-tailed ("greater than") statement, then the P-value is the area to the right of the statistic ω using the appropriate distribution.
- If the alternative hypothesis is a two-tailed ("not equal to") statement, then the P-value is the area outside of the interval $(-\omega, \omega)$ (if ω is positive) or $(\omega, -\omega)$ (if ω is negative). In the case of a χ^2 test, we will have an interval (ω_1, ω_2).

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This would be like concluding that the proportion was not greater than 50% when that was actually the case.

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The confidence of a test is $1 - \alpha$ (the probability of failing to reject a true null hypothesis), and the power of a test is $1 - \beta$ (the probability of rejecting a false null hypothesis).

§8.3 Example (Proportion)

A survey showed that, among the 514 human resource managers polled, 93% said that the appearance of a job applicant was important. Use a 10% significance level to test the claim that at least 95% of human resources managers think that a job applicant's appearance is important.

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$$P(x < -1.882) \leq 0.1.$$

And because $P(x < -1.882) = 0.0299$, we get that we must reject the null hypothesis. In this case, this means that the data does **not** support our original claim!

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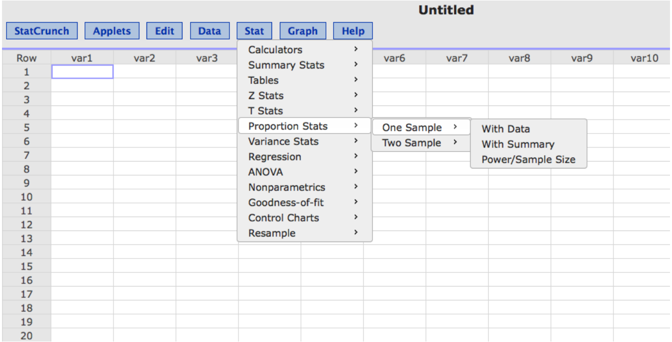

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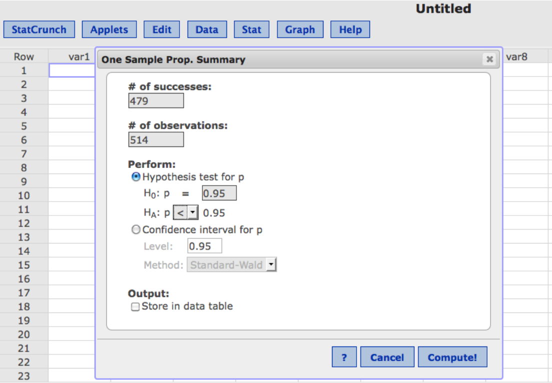

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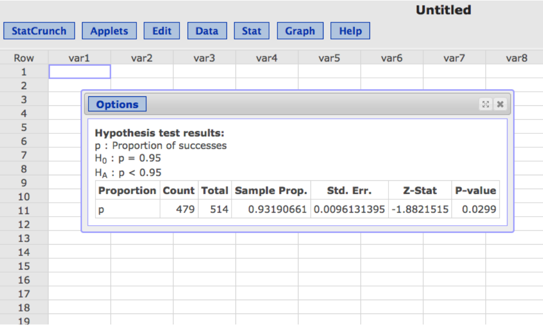

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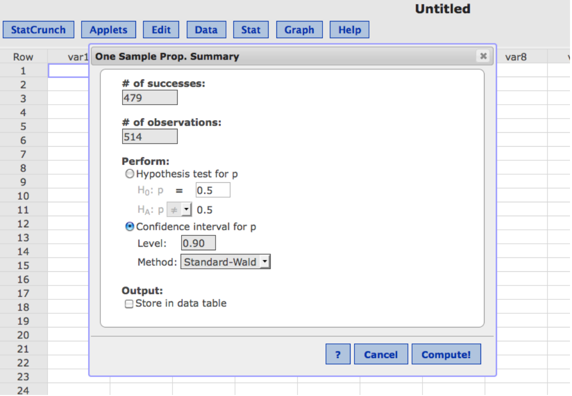

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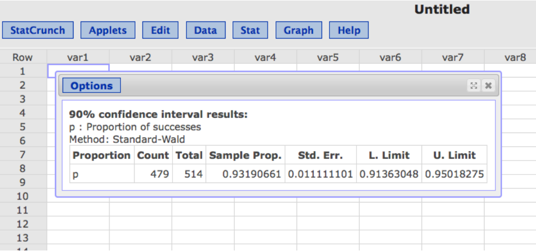

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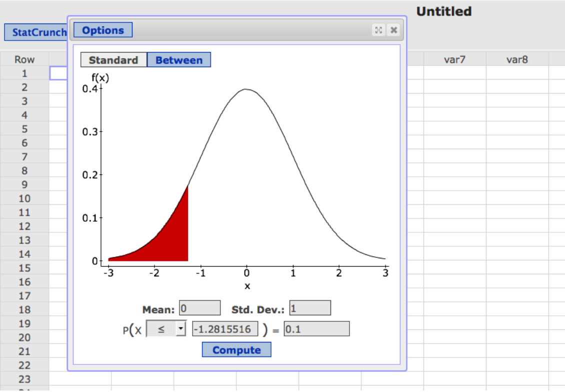

§8.3 Importance of Using an Appropriate Test

Notice the Confidence Interval Method resulting in failing to reject the null hypothesis because it is inherently **two-tailed**. **You must change the significance/confidence level when testing and one-tailed claim!**

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Notice the Confidence Interval Method resulting in failing to reject the null hypothesis because it is inherently **two-tailed**. **You must change the significance/confidence level when testing and one-tailed claim!** We needed a one-tailed test because our alternative hypothesis was one-tailed.

§8.4 Example (Mean)

Use a 0.05 significance level to test the claim that the data below comes from a normally distributed population with mean less than 1.

0.38, 0.55, 1.54, 1.55, 0.50, 0.60, 0.92, 0.96, 1.00, 0.86, 1.46.

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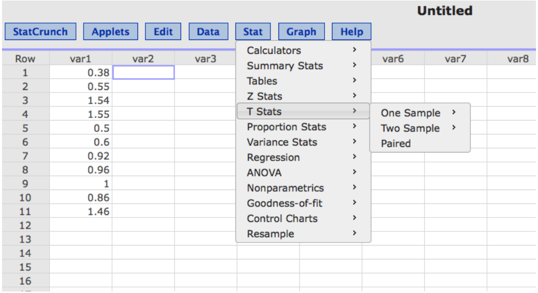

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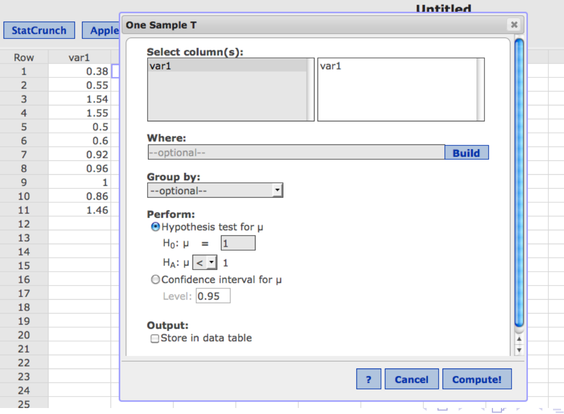

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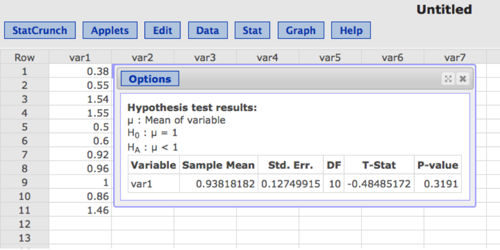

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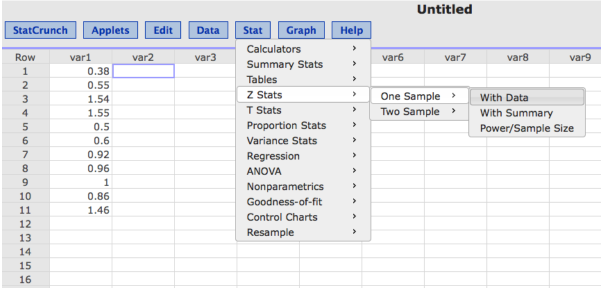

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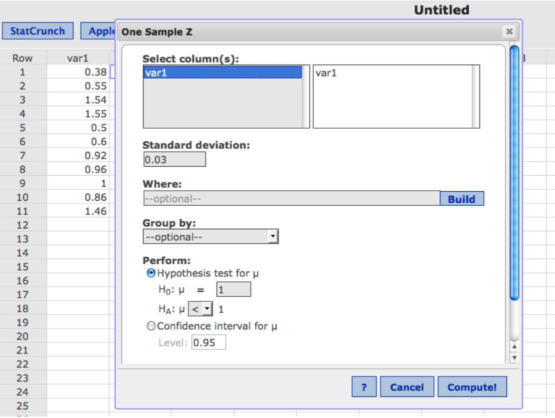

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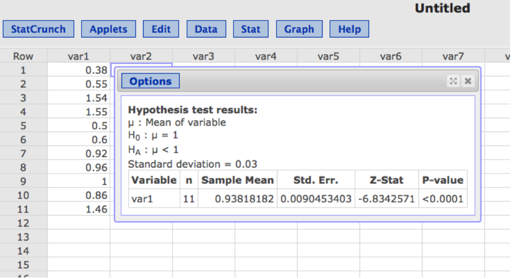

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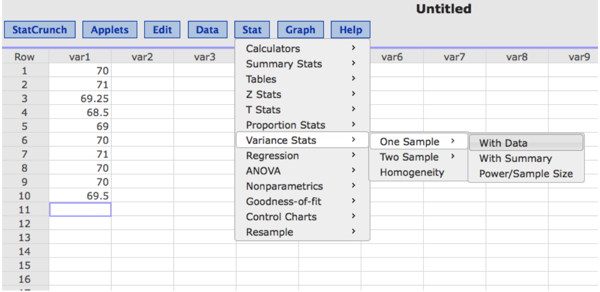

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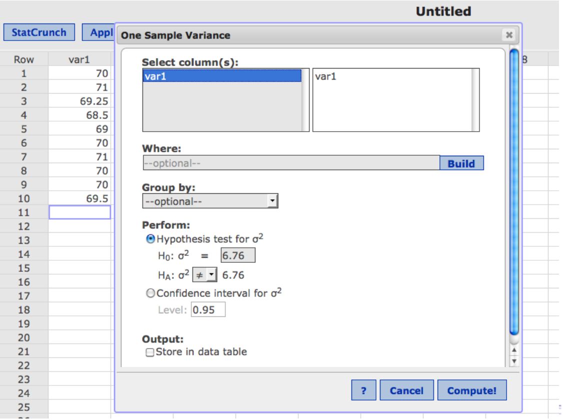

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