Lecture 13:
Chapter 9

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UAB Mathematics

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§9.1 Testing Claims About Two Populations

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- A certain vaccine makes children less likely to contract the measles.
- Women's mean body temperatures are lower than men's.

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$$H_0 : p_1 - p_2 = 0$$
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Z-stat	$\hat{p}_1$ **and** $\hat{p}_2$	$\bar{p}$	E
$\dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$	$\dfrac{x_1}{n_1}$ and $\dfrac{x_2}{n_2}$	$\dfrac{x_1 + x_2}{n_1 + n_2}$	$z_{\frac{\alpha}{2}}\sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}}$

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In a group of 100 people given \$1 bills, 28 spent money. In a group of 121 people given four quarters, 35 spent money. Test the claim that the proportion of people who spent more money when given four quarters is higher than the proportion of people who spent money when given a dollar bill. Use significance $\alpha = 0.1$.

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Thus, we have

$$z = \frac{-0.00926}{\sqrt{\frac{(0.28507)(0.71493)}{100} + \frac{(0.28507)(0.71493)}{121}}} = -0.15182.$$

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Since $z = -0.15182$, we get that out P-value is
$P(X \leq -0.15182) = 0.4396$.

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Since $z = -0.15182$, we get that out P-value is $P(X \leq -0.15182) = 0.4396$. Because the P-value is greater than α, we fail to reject our null hypothesis. This means that there is **not enough evidence to support the claim** that $p_1 < p_2$, i.e. our original claim (that it's more likely for someone to spend money when given four quarters than when given a dollar bill) is not supported by this data.

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Finally, we could test the hypothesis using the critical value method.

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First, note the requirements for these types of hypothesis tests.

- The samples both come from a normally distributed population or are samples with size more than 30.
- Both samples are independent simple random samples.

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If we do not know σ_1 or σ_2 and we assume that they are equal to each other, then we use a T test with **pooled** sample variance.

Researchers conducted a study to determine if olive oil was antibacterial. The first sample consisted of 12 cultures treated with olive oil. The mean number of bacteria in the olive oil cultures was 0.57 million, with a standard deviation of 0.54 million. The second sample consisted of 12 untreated cultures. The mean number of bacteria in the untreated cultures was 0.67 million with standard deviation of 0.89 million. Assume that the populations of bacteria in these cultures in normally distributed.

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$$H_1 : \mu_1 < \mu_2$$

We have

	Sample 1	Sample 2
$\bar{x}$	0.57	0.67
s	0.54	0.89
n	12	12

And we use a T-stat with unpooled variance, getting $t = -0.333$ and $P = 0.3716$.

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And we use a T-stat with unpooled variance, getting $t = -0.333$ and $P = 0.3716$. Thus, we fail to reject the null hypothesis. There is not enough evidence to support the claim that the average number of bacteria in a culture treated with olive oil is less than the average number of bacteria in a culture left untreated.

A candy manufacturer makes blue candies and red candies. They are made on the same machinery which ensures that they vary in size by the same amount. But they are made of different material, so the manufacturer is unsure which is heavier. She suspects they have the same weight. She samples 301 blue candies and 205 red candies, discovering that they have a mean weight of 1.50 grams and 1.40 grams respectively and standard deviations of 0.049 and 0.051 grams respectively. Test this claim with significance $\alpha = 0.01$. Use a confidence interval.

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	Sample 1	Sample 2
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And we use a T-stat with pooled variance, getting the confidence interval (0.089,0.111).

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Make sure that, when doing a two-tailed test via a confidence interval, that you ensure an area of α is to the left and right of the interval!

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The first are independent and the second are dependent.

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- s_d, the standard deviation of these differences for the sample
- n, the number of pairs of sample data

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The following criteria must be met to test claims about the mean of two dependent samples.

- The sample data must be matched pairs.
- The samples must be simple random samples.
- Either the differences must be normally distributed or the number of pairs must be larger than 30.

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We would like to test the claim that presidents tend to be taller than their main opponent, i.e. the taller candidate wins, with a significance level $\alpha = 0.05$. We have the following five pairs of data (whose differences are normally distributed).

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Height O:	170	185	175	180	178

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From StatCrunch, we get the differences.

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Differences:	19	-12	8	0	1

We also get a P-value of 0.2819, which tells us we should fail to reject our null hypothesis. This means our data does not support our original claim that presidents tend to be taller than their opponents.