Lecture 3:
Chapter 3

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UAB Mathematics

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- mean
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- midrange
- weighted mean

Definition (Mean)

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A sample of 10 fun sized candy bags showed that each bag had the following number of candies: 12, 12, 12, 13, 14, 14, 15, 15, 16, 16. What's the average number of candies per bag for this sample?

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$$m = \frac{12 + 12 + 12 + 13 + 14 + 14 + 15 + 15 + 16 + 16}{10} = 13.9.$$

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A sample of 10 fun sized candy bags showed that each bag had the following number of candies: 12, 12, 12, 13, 14, 14, 15, 15, 16, 16. What's the median number of candies in a bag? These are already ordered! Because there is an even number, the median is the average of the 5th and 6th data points, i.e. it is the average of 14 and 14, which is 14.

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- The midrange is the point directly between the lowest and highest data points.

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- A weighted average places greater or smaller significance on certain data points. (An example would be GPA calculation.)

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1, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 9.

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1, 1, 2, 2, 3, 3, 4, 5, 5, 5, 5, 6, 7, 7, 8, 8, 9, 9.

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§3.3 Standard Deviation

Definition (Standard Deviation)

The **standard deviation** of a set of data points is given by

$$s = \sqrt{\dfrac{\sum\limits_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = \sqrt{\dfrac{n\sum\limits_{i=1}^{n}x_i^2 - \left(\sum\limits_{i=1}^{n}x_i\right)^2}{n(n-1)}}$$

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The standard deviation measures how far the data points vary from the **mean**. When is it zero? Is it ever negative?

Example (Calculate Standard Deviation)

Use 1, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 9.

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Use 1, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 9. Well, $\sum\limits_{i=1}^{n} x_i^2 = 492$ and $\bar{x} = 5$, so we have

$$s = \sqrt{\frac{18(492) - 90^2}{18(17)}} \approx 1.57$$

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$$s = \sqrt{\frac{18(568) - 90^2}{18(17)}} \approx 2.63$$

How is the standard deviation useful?

- *When means are similar*, you can use the standard deviation to see differences in variation in samples.
- The standard deviation is less sentitive than range for measuring variation.
- As a general rule of thumb, you should expect to see about 95% of all data points falling within 2 standard deviations of the mean.

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What are the differences between population standard deviation and standard deviation?

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- Variance carries the square of the units of the data it describes.
- Variance is an unbiased estimator while standard deviation is a biased estimator (in the sense described later).
- Variance is always non-negative.

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Theorem (Chebyshev's Theorem)

The proportion of any set of data lying within K standard deviations of the mean is always at least $1 - 1/K^2$, where $K > 1$.

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- $\approx 68\%$ of all values lie within s of m,
- $\approx 95\%$ of all values lie within $2s$ of m, and
- $\approx 99.7\%$ of all values lie within $3s$ of m.

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Definition (Coefficient of Variation)

$$CV = \frac{s}{\bar{x}} \cdot 100\% \quad CV = \frac{\sigma}{\mu} \cdot 100\%$$

Example (Comparing apples and oranges)

Apples weigh an average of 7 ounces with a standard deviation of 2.1 ounces. Oranges have an average volume of 190 mL with a standard deviation of 2.1 mL.

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Example (Comparing apples and oranges)

Apples weigh an average of 7 ounces with a standard deviation of 2.1 ounces. Oranges have an average volume of 190 mL with a standard deviation of 2.1 mL. Because $CV_A \approx 30\%$ whereas $CV_O \approx 1.1\%$, we know that apples vary in weight much more than oranges vary in volume.

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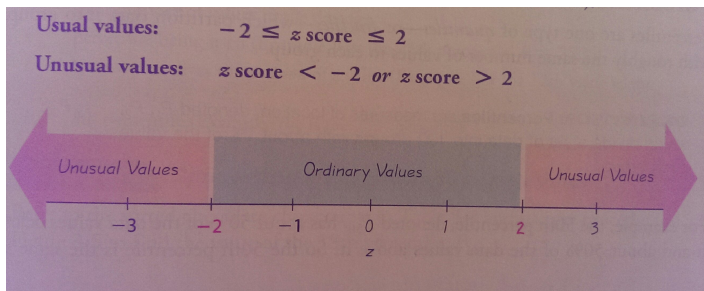How do we tell where a piece of data 'fits' into the larger data set? We use measures of **relative standing**.

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> The z-score for a datum is a how many standard deviations it is above or below the mean.
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> $$z = \frac{x - \bar{x}}{s}, \quad z = \frac{x - \mu}{\sigma}$$

'Typical' values have a z-score of absolute value less than or equal to 2.

Usual values: $-2 \leq z \text{ score} \leq 2$

Unusual values: $z \text{ score} < -2$ *or* $z \text{ score} > 2$

Unusual Values   Ordinary Values   Unusual Values

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

z

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$$\text{percentile value of } x = \frac{\text{number of values less than } x}{\text{total number of values}} \cdot 100$$

§3.4 Percentiles/Quantiles/Quartiles

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$$\text{percentile value of } x = \frac{\text{number of values less than } x}{\text{total number of values}} \cdot 100$$

First, second, and third quartiles are P_{25}, P_{50}, and P_{75}, respectively.

Briefly describe interquartile, semi-interquartile, midquartile, and 10-90 percentile range.

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