Lecture 4:
Chapter 4

C C Moxley

UAB Mathematics

10 June 15

§4.2 Basic Concepts of Probability

Procedure	Event	Simple Event	Sample Space

§4.2 Basic Concepts of Probability

Procedure	Event	Simple Event	Sample Space
rolling a die	6 or 2	6	$\{1, 2, 3, 4, 5, 6\}$

Procedure	Event	Simple Event	Sample Space
rolling a die	6 or 2	6	$\{1, 2, 3, 4, 5, 6\}$
three tests	PPP or FFF	PFP	$\{PPP, PPF, \ldots, FFF\}$

§4.2 Determining Probability of an Event

- **Relative frequency of probability:**

§4.2 Determining Probability of an Event

- **Relative frequency of probability:** This involves experimenting. (Law of Large Numbers)

§4.2 Determining Probability of an Event

- **Relative frequency of probability:** This involves experimenting. (Law of Large Numbers)

$$P(A) \approx \frac{\text{how many times event } A \text{ occurred}}{\text{number of times procedure repeated}}$$

§4.2 Determining Probability of an Event

- **Relative frequency of probability:** This involves experimenting. (Law of Large Numbers)

$$P(A) \approx \frac{\text{how many times event } A \text{ occurred}}{\text{number of times procedure repeated}}$$

- **Classical approach with equally likely outcomes:**

§4.2 Determining Probability of an Event

- **Relative frequency of probability:** This involves experimenting. (Law of Large Numbers)

$$P(A) \approx \frac{\text{how many times event } A \text{ occurred}}{\text{number of times procedure repeated}}$$

- **Classical approach with equally likely outcomes:**

$$P(A) = \frac{\text{number of simple events in } A}{\text{number of possible outcomes}} = \frac{s}{n}$$

§4.2 Determining Probability of an Event

- **Relative frequency of probability:** This involves experimenting. (Law of Large Numbers)

$$P(A) \approx \frac{\text{how many times event } A \text{ occurred}}{\text{number of times procedure repeated}}$$

- **Classical approach with equally likely outcomes:**

$$P(A) = \frac{\text{number of simple events in } A}{\text{number of possible outcomes}} = \frac{s}{n}$$

- **Subjective probabilities:**

§4.2 Determining Probability of an Event

- **Relative frequency of probability:** This involves experimenting. (Law of Large Numbers)

$$P(A) \approx \frac{\text{how many times event } A \text{ occurred}}{\text{number of times procedure repeated}}$$

- **Classical approach with equally likely outcomes:**

$$P(A) = \frac{\text{number of simple events in } A}{\text{number of possible outcomes}} = \frac{s}{n}$$

- **Subjective probabilities:** Estimate $P(A)$ by using knowledge of relevant circumstances.

§4.2 Examples

A survey showed that out of 1010 US adults, 205 smoked. Find the probability that a randomly selected adult smokes in the US.

§4.2 Examples

A survey showed that out of 1010 US adults, 205 smoked. Find the probability that a randomly selected adult smokes in the US.

Use the relative frequency method:

§4.2 Examples

A survey showed that out of 1010 US adults, 205 smoked. Find the probability that a randomly selected adult smokes in the US.

Use the relative frequency method:

$$\frac{205}{1010} \approx 20.3\%$$

§4.2 Examples

What is the chance that I will come to class Thursday with a cane and tophat on?

§4.2 Examples

What is the chance that I will come to class Thursday with a cane and tophat on?

Use the subjective probability method:

What is the chance that I will come to class Thursday with a cane and tophat on?

Use the subjective probability method: Only 1 in 100000 people in the US own both a cane and tophat.

§4.2 Examples

What is the chance that I will come to class Thursday with a cane and tophat on?

Use the subjective probability method: Only 1 in 100000 people in the US own both a cane and tophat. The probability is very small, maybe 0.00001.

§4.2 Examples

What are the changes of drawing a king out of a deck of cards and then drawing another king out of a second deck of cards?

What are the changes of drawing a king out of a deck of cards and then drawing another king out of a second deck of cards?

Use the classical approach.

§4.2 Examples

What are the changes of drawing a king out of a deck of cards and then drawing another king out of a second deck of cards?

Use the classical approach. There are 52^2 possible outcomes, of which 4^2 are drawing a king from the first deck and a king from the second deck.

§4.2 Examples

What are the changes of drawing a king out of a deck of cards and then drawing another king out of a second deck of cards?

Use the classical approach. There are 52^2 possible outcomes, of which 4^2 are drawing a king from the first deck and a king from the second deck. So, the probability is $\frac{16}{2704} \approx 0.59\%$.

§4.2 Definitions

Definition (Complement of an Event)

The complement of an event A, written $\bar{A}$ is all the outcomes in which A does not occur.

§4.2 Definitions

Definition (Complement of an Event)

The complement of an event A, written $\bar{A}$ is all the outcomes in which A does not occur.

Definition (Unusual/Unlikely)

An event is unlikely if the probability of it happening is less than about 5%. An event is unusual if it has an unusually high or low number of outsomes of a particular type, i.e. the number of outcomes of a particular type is far from what we might expect.

§4.2 Definitions

Definition (Complement of an Event)

The complement of an event A, written $\bar{A}$ is all the outcomes in which A does not occur.

Definition (Unusual/Unlikely)

An event is unlikely if the probability of it happening is less than about 5%. An event is unusual if it has an unusually high or low number of outsomes of a particular type, i.e. the number of outcomes of a particular type is far from what we might expect.

Discuss the rare event rule and how it is used to investigate hypotheses.

§4.3 Addition Rule

If you want to determine the chances of an outcome being in event A or event B, use the addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

§4.3 Addition Rule

If you want to determine the chances of an outcome being in event A or event B, use the addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Definition (Mutually Exclusive)

Two events A and B are mutually exclusive if they cannot occur at the same time.

§4.3 Addition Rule

If you want to determine the chances of an outcome being in event A or event B, use the addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Definition (Mutually Exclusive)

Two events A and B are mutually exclusive if they cannot occur at the same time.

Thus, if A and B are mutually exclusive, then $P(A \text{ and } B) =$

§4.3 Addition Rule

If you want to determine the chances of an outcome being in event A or event B, use the addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Definition (Mutually Exclusive)

Two events A and B are mutually exclusive if they cannot occur at the same time.

Thus, if A and B are mutually exclusive, then $P(A \text{ and } B) = 0$.

Example (In a sample of 50 people, 30 had glasses, 35 had contacts, and 23 had both contacts and glasses.)

What is the chance that a randomly selected member of this sample had either contacts or glasses?

§4.3 Addition Rule

If you want to determine the chances of an outcome being in event A or event B, use the addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Definition (Mutually Exclusive)

Two events A and B are mutually exclusive if they cannot occur at the same time.

Thus, if A and B are mutually exclusive, then $P(A \text{ and } B) = 0$.

Example (In a sample of 50 people, 30 had glasses, 35 had contacts, and 23 had both contacts and glasses.)

What is the chance that a randomly selected member of this sample had either contacts or glasses?

$$P(\text{contacts or glasses}) =$$

§4.3 Addition Rule

If you want to determine the chances of an outcome being in event A or event B, use the addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Definition (Mutually Exclusive)

Two events A and B are mutually exclusive if they cannot occur at the same time.

Thus, if A and B are mutually exclusive, then $P(A \text{ and } B) = 0$.

Example (In a sample of 50 people, 30 had glasses, 35 had contacts, and 23 had both contacts and glasses.)

What is the chance that a randomly selected member of this sample had either contacts or glasses?

$$P(\text{contacts or glasses}) = \frac{30}{50} + \frac{35}{50} - \frac{23}{50} = \frac{42}{50}.$$

§4.3 Complementary Events Rule

Because A and $\bar{A}$ are mutually exclusive and because together they make up the whole set of outcomes, we have that

§4.3 Complementary Events Rule

Because A and $\bar{A}$ are mutually exclusive and because together they make up the whole set of outcomes, we have that

$$P(A \text{ or } \bar{A}) = P(A) + P(\bar{A}) - P(A \text{ and } \bar{A}) = P(A) + P(\bar{A}) = 1.$$

Example (Find $P(\overline{A \text{ or } B})$.)

§4.3 Complementary Events Rule

Because A and $\bar{A}$ are mutually exclusive and because together they make up the whole set of outcomes, we have that

$$P(A \text{ or } \bar{A}) = P(A) + P(\bar{A}) - P(A \text{ and } \bar{A}) = P(A) + P(\bar{A}) = 1.$$

Example (Find $P(\overline{A \text{ or } B})$.)

$P(\overline{A \text{ or } B}) = 1 - P(A) - P(B) + P(A \text{ and } B)$.

§4.4 Multiplication Rule

Definition (Independent Events)

Two events A and B are said to be **independent** if the occurance of one event does not affect the probability of the occurance of the other. Otherwise, we call events dependent.

§4.4 Multiplication Rule

Definition (Independent Events)

Two events A and B are said to be **independent** if the occurance of one event does not affect the probability of the occurance of the other. Otherwise, we call events dependent.

Discuss $P(A \text{ and } B)$ and $P(B|A)$.

§4.4 Multiplication Rule

Definition (Independent Events)

Two events A and B are said to be **independent** if the occurance of one event does not affect the probability of the occurance of the other. Otherwise, we call events dependent.

Discuss $P(A \text{ and } B)$ and $P(B|A)$.

The probability that two events occur is equal to the probability that the first occurs times the probability that the second occurs **if these events are independent**!

§4.4 Multiplication Rule

Independent	Dependent
$P(A \text{ and } B) = P(A)P(B)$	$P(A \text{ and } B) = P(A)P(B\|A)$

§4.4 Multiplication Rule

Independent	Dependent
$P(A \text{ and } B) = P(A)P(B)$	$P(A \text{ and } B) = P(A)P(B\|A)$

Example (50 Tests: 10 As, 30 Bs, 5 Cs, 5 Ds)

What is the probability that two randomly selected grades are both Bs?

§4.4 Multiplication Rule

Independent	Dependent
$P(A \text{ and } B) = P(A)P(B)$	$P(A \text{ and } B) = P(A)P(B\|A)$

Example (50 Tests: 10 As, 30 Bs, 5 Cs, 5 Ds)

What is the probability that two randomly selected grades are both Bs? With replacement: $\frac{9}{25}$.

§4.4 Multiplication Rule

Independent	Dependent
$P(A \text{ and } B) = P(A)P(B)$	$P(A \text{ and } B) = P(A)P(B\|A)$

Example (50 Tests: 10 As, 30 Bs, 5 Cs, 5 Ds)

What is the probability that two randomly selected grades are both Bs? With replacement: $\frac{9}{25}$. Without replacement: $\frac{87}{245}$

§4.4 Multiplication Rule

Independent	Dependent
$P(A \text{ and } B) = P(A)P(B)$	$P(A \text{ and } B) = P(A)P(B\|A)$

Example (50 Tests: 10 As, 30 Bs, 5 Cs, 5 Ds)

What is the probability that two randomly selected grades are both Bs? With replacement: $\frac{9}{25}$. Without replacement: $\frac{87}{245}$

Example (What are the chances that 26 randomly chosen people have all different birthdays?)

$$\frac{(365)(364)(363)\ldots(341)(340)}{(365)(365)(365)\ldots(365)(365)} = \frac{(365)(364)(363)\ldots(341)(340)}{365^{26}} \approx 40.18\%$$

§4.4 Multiplication Rule: Redundancy

The multiplication rule for independent events helps illustrate why some important industrial components have redundancy: If an oil pipeline has five different oil pressure measuring tools to ensure that the pipeline is not leaking oil and if each of these tools has a fail rate of 5%, then what is the probability that oil is leaking without being detected?

§4.4 Multiplication Rule: Redundancy

The multiplication rule for independent events helps illustrate why some important industrial components have redundancy: If an oil pipeline has five different oil pressure measuring tools to ensure that the pipeline is not leaking oil and if each of these tools has a fail rate of 5%, then what is the probability that oil is leaking without being detected?

$$0.05^5 = 0.0000003125.$$

It is sometimes usefult to use complements when computing a probability involving the phrase “at least one”.

§4.5 Multiplication Rule: Complements and Conditional Probability

It is sometimes usefult to use complements when computing a probability involving the phrase "at least one". Whenever an event A involves observing "at least one" of some event, it's often easier to compute the complement $\bar{A}$, which is when

§4.5 Multiplication Rule: Complements and Conditional Probability

It is sometimes usefult to use complements when computing a probability involving the phrase "at least one". Whenever an event A involves observing "at least one" of some event, it's often easier to compute the complement $\bar{A}$, which is when **none** of that some event happens!

§4.5 Multiplication Rule: Complements and Conditional Probability

It is sometimes usefult to use complements when computing a probability involving the phrase "at least one". Whenever an event A involves observing "at least one" of some event, it's often easier to compute the complement $\bar{A}$, which is when **none** of that some event happens!

- First, compute $P(\bar{A})$.

§4.5 Multiplication Rule: Complements and Conditional Probability

It is sometimes usefult to use complements when computing a probability involving the phrase "at least one". Whenever an event A involves observing "at least one" of some event, it's often easier to compute the complement $\bar{A}$, which is when **none** of that some event happens!

- First, compute $P(\bar{A})$.
- Then subtract it from 1 because

§4.5 Multiplication Rule: Complements and Conditional Probability

It is sometimes usefult to use complements when computing a probability involving the phrase "at least one". Whenever an event A involves observing "at least one" of some event, it's often easier to compute the complement $\bar{A}$, which is when **none** of that some event happens!

- First, compute $P(\bar{A})$.
- Then subtract it from 1 because $P(A) = 1 - P(\bar{A})$.

§4.5 Example

Now, if we wanted to compute the probability that at least one birthday is shared amongst 26 people (which we will call event A), we can calculate

§4.5 Example

Now, if we wanted to compute the probability that at least one birthday is shared amongst 26 people (which we will call event A), we can calculate

$$1 - 0.4018 = 0.5982.$$

§4.5 Conditional Probability

Definition (Conditional Probability)

A conditional probability of an event is the probability obtained when some additional information is given - particularly that some other event has occurred.

§4.5 Conditional Probability

Definition (Conditional Probability)

A conditional probability of an event is the probability obtained when some additional information is given - particularly that some other event has occurred.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

§4.5 Example

	Positive Test	Negative Test
TB+	15	3
TB-	55	450

§4.5 Example

	Positive Test	Negative Test
TB+	15	3
TB-	55	450

What is the probability that a randomly selected patient had a positive test result (A), given that he is negative for TB (B)?

§4.5 Example

	Positive Test	Negative Test
TB+	15	3
TB-	55	450

What is the probability that a randomly selected patient had a positive test result (A), given that he is negative for TB (B)?

$$P(A|B) = \frac{55}{505} = 0.1089.$$

§4.5 Example

	Positive Test	Negative Test
TB+	15	3
TB-	55	450

What is the probability that a randomly selected patient had a positive test result (A), given that he is negative for TB (B)?

$$P(A|B) = \frac{55}{505} = 0.1089.$$

What is the probability that a randomly selected patient was TB negative (A), given that she had a negative test result?

§4.5 Example

	Positive Test	Negative Test
TB+	15	3
TB-	55	450

What is the probability that a randomly selected patient had a positive test result (A), given that he is negative for TB (B)?

$$P(A|B) = \frac{55}{505} = 0.1089.$$

What is the probability that a randomly selected patient was TB negative (A), given that she had a negative test result?

$$P(A|B) = \frac{450}{453} = 0.9934.$$

§4.5 Example

	Positive Test	Negative Test
TB+	15	3
TB-	55	450

What is the probability that a randomly selected patient had a positive test result (A), given that he is negative for TB (B)?

$$P(A|B) = \frac{55}{505} = 0.1089.$$

What is the probability that a randomly selected patient was TB negative (A), given that she had a negative test result?

$$P(A|B) = \frac{450}{453} = 0.9934.$$

Warning: $P(A|B) \neq P(B|A)$!

§4.6 Counting Rules

1. $m \cdot n =$ the number of ways two events could occur.

§4.6 Counting Rules

1. $m \cdot n =$ the number of ways two events could occur.
2. $n! =$ number of unique permutations of n different items.

§4.6 Counting Rules

1. $m \cdot n =$ the number of ways two events could occur.
2. $n! =$ number of unique permutations of n different items.
3. $\frac{n!}{(n-r)!} =$ number of unique permutations of r items chosen from n items without replacement.

§4.6 Counting Rules

1. $m \cdot n =$ the number of ways two events could occur.
2. $n! =$ number of unique permutations of n different items.
3. $\frac{n!}{(n-r)!} =$ number of unique permutations of r items chosen from n items without replacement.
4. $\frac{n!}{n_1!n_2!\ldots n_k!} =$ number of unique permutations of n items when n_1 are alike, n_2 are alike, $\ldots$, and n_k are alike.

§4.6 Counting Rules

1. $m \cdot n =$ the number of ways two events could occur.
2. $n! =$ number of unique permutations of n different items.
3. $\frac{n!}{(n-r)!} =$ number of unique permutations of r items chosen from n items without replacement.
4. $\frac{n!}{n_1!n_2!...n_k!} =$ number of unique permutations of n items when n_1 are alike, n_2 are alike, . . . , and n_k are alike.
5. $\frac{n!}{(n-r)!r!} =$ number of different combinations of r items chosen without replacement from n different items.

§4.6 Counting Rules

1. How many different ways are there of choosing a card from a deck and rolling a die?

§4.6 Counting Rules

1. How many different ways are there of choosing a card from a deck and rolling a die? $(52)(6) = 312$.

§4.6 Counting Rules

1. How many different ways are there of choosing a card from a deck and rolling a die? $(52)(6) = 312$.
2. How many different ways can you arrange a deck of cards?

§4.6 Counting Rules

1. How many different ways are there of choosing a card from a deck and rolling a die? $(52)(6) = 312$.
2. How many different ways can you arrange a deck of cards? $52!$

§4.6 Counting Rules

1. How many different ways are there of choosing a card from a deck and rolling a die? $(52)(6) = 312$.
2. How many different ways can you arrange a deck of cards? $52!$
3. How many different ways are there of selecting three letters from the alphabet in order?

§4.6 Counting Rules

1. How many different ways are there of choosing a card from a deck and rolling a die? $(52)(6) = 312$.
2. How many different ways can you arrange a deck of cards? $52!$
3. How many different ways are there of selecting three letters from the alphabet in order? $\frac{26!}{23!} = 15600$.

§4.6 Counting Rules

1. How many different ways are there of choosing a card from a deck and rolling a die? $(52)(6) = 312$.
2. How many different ways can you arrange a deck of cards? $52!$
3. How many different ways are there of selecting three letters from the alphabet in order? $\frac{26!}{23!} = 15600$.
4. How many different ordering are there for a, a, b, b, c?

§4.6 Counting Rules

1. How many different ways are there of choosing a card from a deck and rolling a die? $(52)(6) = 312$.
2. How many different ways can you arrange a deck of cards? $52!$
3. How many different ways are there of selecting three letters from the alphabet in order? $\frac{26!}{23!} = 15600$.
4. How many different ordering are there for a, a, b, b, c? $\frac{5!}{2!2!1!} = 30$.

§4.6 Counting Rules

1. How many different ways are there of choosing a card from a deck and rolling a die? $(52)(6) = 312$.
2. How many different ways can you arrange a deck of cards? $52!$
3. How many different ways are there of selecting three letters from the alphabet in order? $\frac{26!}{23!} = 15600$.
4. How many different ordering are there for a, a, b, b, c? $\frac{5!}{2!2!1!} = 30$.
5. How many different ways are there of chosing four letters from the first six letters of the alphabet if order doesn't matter?

§4.6 Counting Rules

1. How many different ways are there of choosing a card from a deck and rolling a die? $(52)(6) = 312$.
2. How many different ways can you arrange a deck of cards? $52!$
3. How many different ways are there of selecting three letters from the alphabet in order? $\frac{26!}{23!} = 15600$.
4. How many different ordering are there for a, a, b, b, c? $\frac{5!}{2!2!1!} = 30$.
5. How many different ways are there of chosing four letters from the first six letters of the alphabet if order doesn't matter? $\frac{6!}{4!2!} = 15$.