Lecture 9: Chapter 7

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§7.1 Inferential Statistics

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Definition (Inferential Statistics)

The use of sample data to make numerical statements about the statistical/probabilistic characteristics of a population.

§7.2 Estimating a Population Proportion

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A **point estimate** is a single value used to estimate a population parameter.

E.g. 35% (or 0.35) is a point estimate.

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E.g. 95% (or 0.95) is a confidence level. In this case, $\alpha = 0.05$. Often, you are asked to create the confidence interval given a certain desired confidence level. In this chapter, we use confidence intervals for informal hypothesis testing.

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A **critical value** is the number on the boarder which separates typical from atypical values. The number $z_{\frac{\alpha}{2}}$ us a critical value which separates an area of $\frac{\alpha}{2}$ in the right tail of the standard normal distribution.

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Different distributions must have critical values calculated differently!

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Definition (Margin of Error)

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You can also use this information to calculate how big a sample size needs to be to yield a certain level of confidence with a certain error size. (Always round up when doing this.)

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which implies that $n \approx 3269.78$, so we need a sample size of at least 3268.

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- There are at least five successes and five failures.

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$$n - 1,$$

where n is the sample size.

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which means our sample size must be at least 133 cats.

§7.4 Estimating Population Standard Deviation or Variance

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§7.4 The χ^2 Distribution

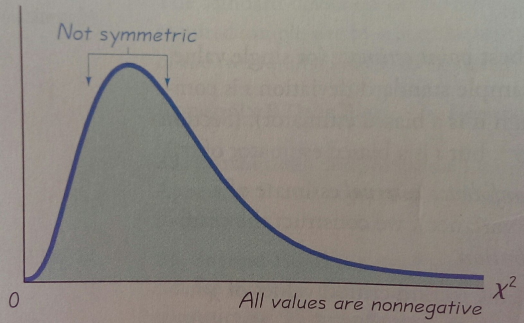

Figure 7-7 Chi-Square Distribution

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$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}},$$

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- The population **must be normally distributed, even if the sample is large!**

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The time a student spends on a section of the SAT is given in minutes: 25, 21, 21, 18, 13, 13, 14, 16, 12, 22, 25, 25, 30. These times are known to be normally distributed. Calculate a 90% confidence interval for the standard deviation of these times.

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We have that $\overline{x} = 19.62$ and $s = 5.752$.

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Therefore,

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}} \implies$$

$$4.35 = \sqrt{\frac{(12)5.752^2}{21.026}} < \sigma < \sqrt{\frac{(12)5.752^2}{5.226}} = 8.68.$$

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So we have that the 90% confidence interval is (4.35,8.68). What does this actually mean?

This means that if we took a large number of simple random samples of size 13 and calculated their standard deviation, then 90% of the time, your calculated standard deviation would fall into the interval (4.35,8.68). It does not mean that there is a 90% chance that σ falls within the interval.

§7.4 Precise Language

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What does it mean to have a 95% CI for population mean? It is precisely an interval which, when a random selection is made from the sampling distribution of the mean, will contain the given selection 95% of the time. We'll talk about how this relates to hypothesis testing in the next chapter, and it will be much easier to speak about what a 95% CI (or confidence level) means in the context of hypothesis testing.