# Lesson 12: Chapter 7 Sections 2-3

Caleb Moxley

**BSC Mathematics** 

14 October 15

We've already discussed evaluating claims based on paired data — the mean of the differences.

We've already discussed evaluating claims based on paired data — the mean of the differences. Now, we'll discuss inference for claims involving two different populations — the difference of means!

We've already discussed evaluating claims based on paired data — the mean of the differences. Now, we'll discuss inference for claims involving two different populations — the difference of means! We will be able to answer questions like the following.

We've already discussed evaluating claims based on paired data — the mean of the differences. Now, we'll discuss inference for claims involving two different populations — the difference of means! We will be able to answer questions like the following.

Is the average number of visits to a store by consumers higher or lower if the store is having a sale versus having a grand opening?

We've already discussed evaluating claims based on paired data — the mean of the differences. Now, we'll discuss inference for claims involving two different populations — the difference of means! We will be able to answer questions like the following.

- Is the average number of visits to a store by consumers higher or lower if the store is having a sale versus having a grand opening?
- Do women or men have higher average blood pressure?

We've already discussed evaluating claims based on paired data — the mean of the differences. Now, we'll discuss inference for claims involving two different populations — the difference of means! We will be able to answer questions like the following.

- Is the average number of visits to a store by consumers higher or lower if the store is having a sale versus having a grand opening?
- Do women or men have higher average blood pressure?
- Are the average monthly balances on credit cards different for card holders with high lines of credit or low lines of credit?

When we're performing inference on two sample means, we assume that

the variable separating the two populations is an explanatory variable,

- the variable separating the two populations is an explanatory variable,
- the variable being measured is a response variable,

- the variable separating the two populations is an explanatory variable,
- the variable being measured is a response variable,
- each sample is from a distinct population, and

- the variable separating the two populations is an explanatory variable,
- the variable being measured is a response variable,
- each sample is from a distinct population, and
- the responses in each group are independent of those in the other group.

When we're performing inference on two sample means, we assume that

- the variable separating the two populations is an explanatory variable,
- the variable being measured is a response variable,
- each sample is from a distinct population, and
- the responses in each group are independent of those in the other group.

We may or may not assume that we know  $\sigma s$  for the populations.

### Definition (two-sample z statistic)

Suppose  $\bar{x}_1$  and  $\bar{x}_2$  are the means of two SRSs drawn from populations with distributions  $N(\mu_1, \sigma_1)$  and  $N(\mu_2, \sigma_2)$  respectively. Then the two-sample z statistic

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

has the standard normal sampling distribution.

### Definition (two-sample z statistic)

Suppose  $\bar{x}_1$  and  $\bar{x}_2$  are the means of two SRSs drawn from populations with distributions  $N(\mu_1, \sigma_1)$  and  $N(\mu_2, \sigma_2)$  respectively. Then the two-sample z statistic

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

has the standard normal sampling distribution.

This z statistic can be used to compute P-values or compared to critical values using the standard normal tables or Minitab.

### Definition (two-sample z statistic)

Suppose  $\bar{x}_1$  and  $\bar{x}_2$  are the means of two SRSs drawn from populations with distributions  $N(\mu_1, \sigma_1)$  and  $N(\mu_2, \sigma_2)$  respectively. Then the two-sample z statistic

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

has the standard normal sampling distribution.

This z statistic can be used to compute P-values or compared to critical values using the standard normal tables or Minitab. Here's the thing: We practically never know  $\sigma_1$  and/or  $\sigma_2$ . Why?

### Definition (two-sample z statistic)

Suppose  $\bar{x}_1$  and  $\bar{x}_2$  are the means of two SRSs drawn from populations with distributions  $N(\mu_1, \sigma_1)$  and  $N(\mu_2, \sigma_2)$  respectively. Then the two-sample z statistic

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

has the standard normal sampling distribution.

This z statistic can be used to compute P-values or compared to critical values using the standard normal tables or Minitab. Here's the thing: We practically never know  $\sigma_1$  and/or  $\sigma_2$ . Why? So we usually use the t statistic corresponding to the z statistic above.

### Definition (two-sample *t* statistic)

Suppose  $\bar{x}_1$  and  $\bar{x}_2$  are the means of two SRSs drawn from populations with normal distribution with unknown standard deviations (which are assumed not to be equal) and means  $\mu_1$  and  $\mu_2$  respectively. Then the two-sample t statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

has a t(k) distribution where k is calculated in one of the two methods below.

### Definition (two-sample *t* statistic)

Suppose  $\bar{x}_1$  and  $\bar{x}_2$  are the means of two SRSs drawn from populations with normal distribution with unknown standard deviations (which are assumed not to be equal) and means  $\mu_1$  and  $\mu_2$  respectively. Then the two-sample t statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

has a t(k) distribution where k is calculated in one of the two methods below.

$$k = \min(n_1 - 1, n_2 - 1) \text{ or } k = \left[ \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1} \left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1} \left(\frac{s_2^2}{n_2}\right)^2} \right]$$

#### Definition (two-sample t confidence interval)

Suppose  $\bar{x}_1$  and  $\bar{x}_2$  are the means of two SRSs drawn from populations with normal distribution with unknown standard deviations (which are assumed not to be equal) and means  $\mu_1$  and  $\mu_2$  respectively. Then the two-sample t C-confidence interval for  $\mu_1 - \mu_2$  is

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where  $t^*$  is the value for the t(k) density curve with area C between  $-t^*$  and  $t^*$ .

#### Definition (two-sample t confidence interval)

Suppose  $\bar{x}_1$  and  $\bar{x}_2$  are the means of two SRSs drawn from populations with normal distribution with unknown standard deviations (which are assumed not to be equal) and means  $\mu_1$  and  $\mu_2$  respectively. Then the two-sample t C-confidence interval for  $\mu_1 - \mu_2$  is

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where  $t^*$  is the value for the t(k) density curve with area C between  $-t^*$  and  $t^*$ .

The k degrees of freedom is calculated as in the previous slide.

Naturally, there's a critical value test for comparing two means. You simply compare the t test statistic for two means with the corresponding  $t^*$  and determine whether the t test statistic is more extreme than  $t^*$  or not.

Naturally, there's a critical value test for comparing two means. You simply compare the t test statistic for two means with the corresponding  $t^*$  and determine whether the t test statistic is more extreme than  $t^*$  or not.

In order to conduct any of these tests, though, you must have one of the following conditions met:

Naturally, there's a critical value test for comparing two means. You simply compare the t test statistic for two means with the corresponding  $t^*$  and determine whether the t test statistic is more extreme than  $t^*$  or not.

In order to conduct any of these tests, though, you must have one of the following conditions met:

■ If  $n_1 + n_2 < 15$ , only use the test if the sample is quite normal.

Naturally, there's a critical value test for comparing two means. You simply compare the t test statistic for two means with the corresponding  $t^*$  and determine whether the t test statistic is more extreme than  $t^*$  or not.

In order to conduct any of these tests, though, you must have one of the following conditions met:

- If  $n_1 + n_2 < 15$ , only use the test if the sample is quite normal.
- If  $15 \le n_1 + n_2 < 40$ , you may use the test except in the presence of strong outliers/skewness.

Naturally, there's a critical value test for comparing two means. You simply compare the t test statistic for two means with the corresponding  $t^*$  and determine whether the t test statistic is more extreme than  $t^*$  or not.

In order to conduct any of these tests, though, you must have one of the following conditions met:

- If  $n_1 + n_2 < 15$ , only use the test if the sample is quite normal.
- If  $15 \le n_1 + n_2 < 40$ , you may use the test except in the presence of strong outliers/skewness.
- If  $40 \le n_1 + n_2$ , you can always use the test.

There's one final caveat we need to discuss before examples!

There's one final caveat we need to discuss before examples! In some rare instances, it makes sense to assume that the two population standard deviations are equal.

There's one final caveat we need to discuss before examples! In some rare instances, it makes sense to assume that the two population standard deviations are equal. In this case, the standard deviation of the sampling distribution of the differences of means is different than before.

There's one final caveat we need to discuss before examples! In some rare instances, it makes sense to assume that the two population standard deviations are equal. In this case, the standard deviation of the sampling distribution of the differences of means is different than before. We use

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

instead, which we call the pooled variance of the samples, to calculate the standard deviation of the sampling distribution of the differences of means. We get the following confidence intervals/t test statistics.

### Definition (two-sample t statistic with pooled variances)

Suppose  $\bar{x}_1$  and  $\bar{x}_2$  are the means of two SRSs drawn from populations with normal distribution with unknown standard deviations (which are assumed to be equal) and means  $\mu_1$  and  $\mu_2$  respectively. Then the two-sample t statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

has a t(k) distribution where  $k = n_1 + n_2 - 2$ .

### Definition (two-sample t statistic with pooled variances)

Suppose  $\bar{x}_1$  and  $\bar{x}_2$  are the means of two SRSs drawn from populations with normal distribution with unknown standard deviations (which are assumed to be equal) and means  $\mu_1$  and  $\mu_2$  respectively. Then the two-sample t statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

has a t(k) distribution where  $k = n_1 + n_2 - 2$ .

### Definition (two-sample *t* confidence interval with pooled variances)

Suppose  $\bar{x}_1$  and  $\bar{x}_2$  are the means of two SRSs drawn from populations with normal distribution with unknown standard deviations (which are assumed to be equal) and means  $\mu_1$  and  $\mu_2$  respectively. Then the two-sample t C-confidence interval for  $\mu_1 - \mu_2$  is

$$(\bar{x}_1 - \bar{x}_2) \pm t^* s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where  $t^*$  is the value for the t(k) density curve with area C between  $-t^*$  and  $t^*$  and  $k = n_1 + n_2 - 2$ .

### Example (confidence interval for $\mu_1 - \mu_2$ , unpooled)

We have two samples in the data set below. One column is a sample of heights of men. The other is a sample of heights of women. Assume  $\sigma_1 \neq \sigma_2$ .

Data Set 1

### Example (confidence interval for $\mu_1 - \mu_2$ , unpooled)

We have two samples in the data set below. One column is a sample of heights of men. The other is a sample of heights of women. Assume  $\sigma_1 \neq \sigma_2$ .

#### Data Set 1

If  $\mu_1$  corresponds to the mean of the heights of men and  $\mu_2$  corresponds to the mean of the heights of women, create a 90% CI for  $\mu_1-\mu_2$ .

### Example (confidence interval for $\mu_1 - \mu_2$ , unpooled)

We have two samples in the data set below. One column is a sample of heights of men. The other is a sample of heights of women. Assume  $\sigma_1 \neq \sigma_2$ .

#### Data Set 1

If  $\mu_1$  corresponds to the mean of the heights of men and  $\mu_2$  corresponds to the mean of the heights of women, create a 90% CI for  $\mu_1-\mu_2$ . Use this CI to test the claim that men are taller than women.

### Example (confidence interval for $\mu_1 - \mu_2$ , unpooled)

We have two samples in the data set below. One column is a sample of heights of men. The other is a sample of heights of women. Assume  $\sigma_1 \neq \sigma_2$ .

#### Data Set 1

If  $\mu_1$  corresponds to the mean of the heights of men and  $\mu_2$  corresponds to the mean of the heights of women, create a 90% CI for  $\mu_1-\mu_2$ . Use this CI to test the claim that men are taller than women.

Answer: (9.8,15.8) using technology and k = 77. (9.73,15.84) using tables and k = 39.

#### Example (confidence interval for $\mu_1 - \mu_2$ , unpooled)

We have two samples in the data set below. One column is a sample of heights of men. The other is a sample of heights of women. Assume  $\sigma_1 \neq \sigma_2$ .

#### Data Set 1

If  $\mu_1$  corresponds to the mean of the heights of men and  $\mu_2$  corresponds to the mean of the heights of women, create a 90% CI for  $\mu_1-\mu_2$ . Use this CI to test the claim that men are taller than women.

Answer: (9.8,15.8) using technology and k = 77. (9.73,15.84) using tables and k = 39. Thus, we reject  $H_0$  and support our claim.

### Example (confidence interval for $\mu_1 - \mu_2$ , pooled)

We have two samples in the data set below. One column is a sample of heights of men. The other is a sample of heights of women. Assume  $\sigma_1 = \sigma_2$ .

Data Set 1

#### Example (confidence interval for $\mu_1 - \mu_2$ , pooled)

We have two samples in the data set below. One column is a sample of heights of men. The other is a sample of heights of women. Assume  $\sigma_1 = \sigma_2$ .

#### Data Set 1

If  $\mu_1$  corresponds to the means of the heights of men and  $\mu_2$  corresponds to the mean of the heights of women, create a 90% CI for  $\mu_1-\mu_2$ .

### Example (confidence interval for $\mu_1 - \mu_2$ , pooled)

We have two samples in the data set below. One column is a sample of heights of men. The other is a sample of heights of women. Assume  $\sigma_1 = \sigma_2$ .

#### Data Set 1

If  $\mu_1$  corresponds to the means of the heights of men and  $\mu_2$  corresponds to the mean of the heights of women, create a 90% CI for  $\mu_1 - \mu_2$ . Use this CI to test the claim that men are taller than women.

#### Example (confidence interval for $\mu_1 - \mu_2$ , pooled)

We have two samples in the data set below. One column is a sample of heights of men. The other is a sample of heights of women. Assume  $\sigma_1 = \sigma_2$ .

#### Data Set 1

If  $\mu_1$  corresponds to the means of the heights of men and  $\mu_2$  corresponds to the mean of the heights of women, create a 90% CI for  $\mu_1-\mu_2$ . Use this CI to test the claim that men are taller than women.

Answer: (9.82,15.76) using technology and k=83. (9.82,15.77) using tables and k=83.

#### Example (confidence interval for $\mu_1 - \mu_2$ , pooled)

We have two samples in the data set below. One column is a sample of heights of men. The other is a sample of heights of women. Assume  $\sigma_1 = \sigma_2$ .

#### Data Set 1

If  $\mu_1$  corresponds to the means of the heights of men and  $\mu_2$  corresponds to the mean of the heights of women, create a 90% CI for  $\mu_1-\mu_2$ . Use this CI to test the claim that men are taller than women.

Answer: (9.82,15.76) using technology and k=83. (9.82,15.77) using tables and k=83. Thus, we reject  $H_0$  and support our claim.

#### Example (*P*-value test for two means, unpooled)

We want to test the claim that the average monthly sales at Store A exceed average monthly sales at Store B. We have the following data. Assume that each store has roughly normal monthly sales with different standard deviations.

#### Example (*P*-value test for two means, unpooled)

We want to test the claim that the average monthly sales at Store A exceed average monthly sales at Store B. We have the following data. Assume that each store has roughly normal monthly sales with different standard deviations. Use  $\alpha=0.02$ .

Store	$\bar{x}$	n	s
Store A	15000	20	3200
Store B	14000	10	1500

#### Example (*P*-value test for two means, unpooled)

We want to test the claim that the average monthly sales at Store A exceed average monthly sales at Store B. We have the following data. Assume that each store has roughly normal monthly sales with different standard deviations. Use  $\alpha=0.02$ .

Store	$\bar{x}$	n	S
Store A	15000	20	3200
Store B	14000	10	1500

Well, the *P*-value is 0.127 using technology and k = 27.

#### Example (*P*-value test for two means, unpooled)

We want to test the claim that the average monthly sales at Store A exceed average monthly sales at Store B. We have the following data. Assume that each store has roughly normal monthly sales with different standard deviations. Use  $\alpha=0.02$ .

Store	$\bar{x}$	n	S
Store A	15000	20	3200
Store B	14000	10	1500

Well, the *P*-value is 0.127 using technology and k=27. And *P*-value is somewhere between 0.15 and 0.1 using Table D and k=9.

#### Example (*P*-value test for two means, unpooled)

We want to test the claim that the average monthly sales at Store A exceed average monthly sales at Store B. We have the following data. Assume that each store has roughly normal monthly sales with different standard deviations. Use  $\alpha=0.02$ .

Store	$\bar{x}$	n	S
Store A	15000	20	3200
Store B	14000	10	1500

Well, the P-value is 0.127 using technology and k=27. And P-value is somewhere between 0.15 and 0.1 using Table D and k=9. Thus, we fail to reject  $H_0$  and do not support our claim.

#### Example (P-value test for two means, pooled)

We want to test the claim that the average monthly sales at Store A exceed average monthly sales at Store B. We have the following data. Assume that each store has roughly normal monthly sales with the same standard deviation.

#### Example (P-value test for two means, pooled)

We want to test the claim that the average monthly sales at Store A exceed average monthly sales at Store B. We have the following data. Assume that each store has roughly normal monthly sales with the same standard deviation. Use  $\alpha=0.02$ .

Store	$\bar{x}$	n	s
Store A	15000	20	3200
Store B	14000	10	1500

#### Example (P-value test for two means, pooled)

We want to test the claim that the average monthly sales at Store A exceed average monthly sales at Store B. We have the following data. Assume that each store has roughly normal monthly sales with the same standard deviation. Use  $\alpha=0.02$ .

Store	$\bar{x}$	n	s
Store A	15000	20	3200
Store B	14000	10	1500

Well, the *P*-value is 0.1796 using technology and k = 28.

#### Example (P-value test for two means, pooled)

We want to test the claim that the average monthly sales at Store A exceed average monthly sales at Store B. We have the following data. Assume that each store has roughly normal monthly sales with the same standard deviation. Use  $\alpha=0.02$ .

Store	$\bar{x}$	n	s
Store A	15000	20	3200
Store B	14000	10	1500

Well, the *P*-value is 0.1796 using technology and k=28. And *P*-value is somewhere between 0.2 and 0.15 using Table D and k=28.

#### Example (P-value test for two means, pooled)

We want to test the claim that the average monthly sales at Store A exceed average monthly sales at Store B. We have the following data. Assume that each store has roughly normal monthly sales with the same standard deviation. Use  $\alpha = 0.02$ .

Store	$\bar{x}$	n	s
Store A	15000	20	3200
Store B	14000	10	1500

Well, the P-value is 0.1796 using technology and k=28. And P-value is somewhere between 0.2 and 0.15 using Table D and k=28. Thus, we fail to reject  $H_0$  and do not support our claim.

We may be interested in knowing whether or not it makes sense to pool variances, i.e. to determine if  $\sigma_1^2 = \sigma_2^2$  or not. Luckily, we have a test for this!

We may be interested in knowing whether or not it makes sense to pool variances, i.e. to determine if  $\sigma_1^2 = \sigma_2^2$  or not. Luckily, we have a test for this! Unluckily, however, this test is **extremely sensitive** to non-normality! It only works when both standard deviations are from samples whose underlying populations are normally distributed!

#### Definition (F statistic, F test for variance, and F distributions)

When testing the  $H_0$ :  $\sigma_1^2 = \sigma_2^2$  against  $H_a$ :  $\sigma_1^2 \neq \sigma_2^2$ , compare the F-statistic

$$F = \frac{s_{\text{large}}^2}{s_{\text{small}}^2}.$$

Obtain the P-value for this test by finding the P-value for the  $F^*$  corresponding to F from Table E (or using technology) such that  $F^*$  is the smallest critical value still larger than F. Then double the P-value for  $F^*$  to get a lower estimate for the P-value of the two-sides F-test. Make sure to use the  $F(n_n-1,n_d-1)$  distribution, where  $n_n$  is the number of things in the numerator standard deviation's sample and  $n_d$  is the number of things in the denominator standard deviation's sample.

#### Definition (F statistic, F test for variance, and F distributions)

When testing the  $H_0$ :  $\sigma_1^2 = \sigma_2^2$  against  $H_a$ :  $\sigma_1^2 \neq \sigma_2^2$ , compare the F-statistic

$$F = \frac{s_{\text{large}}^2}{s_{\text{small}}^2}.$$

Obtain the P-value for this test by finding the P-value for the  $F^*$  corresponding to F from Table E (or using technology) such that  $F^*$  is the smallest critical value still larger than F. Then double the P-value for  $F^*$  to get a lower estimate for the P-value of the two-sides F-test. Make sure to use the  $F(n_n-1,n_d-1)$  distribution, where  $n_n$  is the number of things in the numerator standard deviation's sample and  $n_d$  is the number of things in the denominator standard deviation's sample.

You can do the one-sided test by not doubling the *P*-value.



#### Example (two similar standard deviations)

Sample A had a standard deviation of 8.5 with a sample of size 13, and Sample B had a standard deviation of 7.2 with a sample size of 11. Test the claim that the standard deviations of the populations from which these samples are drawn are the same. Assume both underlying populations are normal. Use 10% significance.

#### Example (two similar standard deviations)

Sample A had a standard deviation of 8.5 with a sample of size 13, and Sample B had a standard deviation of 7.2 with a sample size of 11. Test the claim that the standard deviations of the populations from which these samples are drawn are the same. Assume both underlying populations are normal. Use 10% significance.

Answer: F statistic is 1.394. We use the F(12,10) distribution and see that the critical value  $F^*=2.28$  is the smallest critical value larger than F.

#### Example (two similar standard deviations)

Sample A had a standard deviation of 8.5 with a sample of size 13, and Sample B had a standard deviation of 7.2 with a sample size of 11. Test the claim that the standard deviations of the populations from which these samples are drawn are the same. Assume both underlying populations are normal. Use 10% significance.

Answer: F statistic is 1.394. We use the F(12, 10) distribution and see that the critical value  $F^* = 2.28$  is the smallest critical value larger than F. It's P-value is 0.1, so the P-value for this test is at least 0.2, which is larger than our significance!

#### Example (two similar standard deviations)

Sample A had a standard deviation of 8.5 with a sample of size 13, and Sample B had a standard deviation of 7.2 with a sample size of 11. Test the claim that the standard deviations of the populations from which these samples are drawn are the same. Assume both underlying populations are normal. Use 10% significance.

Answer: F statistic is 1.394. We use the F(12,10) distribution and see that the critical value  $F^*=2.28$  is the smallest critical value larger than F. It's P-value is 0.1, so the P-value for this test is at least 0.2, which is larger than our significance! Thus, we fail to reject our null hypothesis and support our claim.

For the previous test, we must assume that the underlying distributions of the populations are normal! Departures from normality are not allowed!