Lesson 13:
Chapter 8 Section 1

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BSC Mathematics

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§8.1 Inference for a Single Proportion

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$$\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right) = N\left(\mu_{\hat{p}}, \sigma_{\hat{p}}\right).$$

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Definition (large n confidence interval for a population proportion)

Choose a SRS of size n from a large population with unknown proportion of success p. The sample proportion is $\hat{p} = \frac{X}{n}$, where X is the number of successes. The standard error of $\hat{p}$ is $\mathrm{SE}_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/n}$, and the margin of error for confidence C is $m = z^* SE_{\hat{p}}$, where z^* is the value of the $N(0,1)$ RV with area C between $-z^*$ and z^*. The approximate level C confidence interval for p is

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In order to get this confidence interval, the number of successes and failures should be at least ten each.

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4. Create the confidence interval $\hat{p} \pm z^* \mathrm{SE}_{\hat{p}}$.

Example

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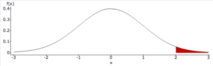
Definition (large sample z test for population proportion)

To test the hypothesis $H_0 : p = p_0$ based on a SRS of size n from a population with unknown p, compute the **test statistic**

$$z = \frac{\hat{p} - p_0}{\sqrt{(p_0(1 - p_0))/n}}.$$

For a $N(0, 1)$ RV, the P-value for a test of H_0 against

$H_a : p > p_0$ is $P(Z > z)$



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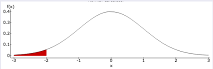
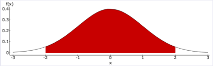


$H_a : p \neq p_0$ is $2P(Z > |z|)$



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Notice: The P-value test given in the previous slide follows essentially the same process as our previous P-value tests! The only difference is the way that we calculated the test statistic and P value.

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- The population from which the sample is drawn should be large or the sample should be done with replacement.
- The sample itself should be large, but not more than 5% of the population.
- There should be at least ten failures and ten successes in the sample.

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Example (pizzas delivered on time)

A pizza delivery chain wants to test the claim that at least 95% of pizzas ordered for delivery at its restaurants are delivered on time. It wants to test this claim with 99% confidence. After taking a SRS of 300 deliveries, it determined that 284 were delivered on time. Does this sample support the claim?

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Answer: Well, $z = -0.265$.

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Answer: Well, $z = -0.265$. And $P(Z < -0.265) = 0.3955$. So we fail to reject H_0 and support our claim.

Example (voters close to polling stations)

A SRS of 2000 voters in Alabama revealed that 1729 lived within 15 minutes of their polling stations. Does this sample support the claim that the proportion of voters in Alabama who live within 15 minutes of their polling stations is 84%? Use $\alpha = 0.02$ and a P-value test.

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Answer: Well, $z = 2.989$. And $2(P(Z > |2.989|) = 2(1 - 0.9986) = 0.0028$. Thus we reject H_0 and fail to support our claim.

Example (loaded coin)

A coin is tossed 200 times, and it lands on heads 115 times. Test the claim that the coin is loaded so that it comes up heads more than 50% of the time. Use a P-value test and a 10% significance.

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Answer: Well, $z = 2.121$. And $P(Z > 2.989) = 0.0169$.

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Answer: Well, $z = 2.121$. And $P(Z > 2.989) = 0.0169$. Thus we reject H_0 and support our claim.

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Make sure you reach the same conclusion using these critical value tests — and remember to compare z and z^* in the appropriate way.

Finally, we'll determine how large we need to make our samples in order to get a confidence interval for a proportion with a particular level of confidence and a particular margin of error!

Definition (sample size n for a desired margin of error)

The level C confidence interval for a proportion p will have a margin of error approximately equal to a specified value m when the sample size satisfies

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where p^* is some guessed proportion of successes for the future sample. If you have no good guess, let p^* be 50%. This maximizes the above function over all p^*, so you can't go wrong with that guess. The formula then becomes

$$n = \left(\frac{z^*}{m}\right)^2 \frac{1}{4}.$$

Example (loaded coin)

We think a coin has been loaded so that it will turn up heads 75% of the time. How large should our sample be in order to create a 95% CI for the proportion of times a coin lands on heads if we want the margin of error to be less than 0.05?

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We would have needed to flip it 385 times.