Lesson 14: Chapter 8 Section 2

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BSC Mathematics

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§8.2 Comparing Two Proportions

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§8.2 Comparing Two Proportions

Let's see why the mean and standard deviation of the distribution for D are

$$p_1 - p_2 \text{ and } \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}.$$

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Of course, we don't know p_1 or p_2, so we approximate this with the estimates $\hat{p}_1$ and $\hat{p}_2$. And we get all our resulting methods!

§8.2 Comparing Two Proportions

Definition (large samples confidence interval for two proportions)

Choose a SRS of size n_1 from a large population with unknown proportion of success p_1 and a SRS of size n_2 from a large population with unknown proportion of success p_2. The estimate for the difference in proportions is $D = \hat{p}_1 - \hat{p}_2$. The standard error of D is

$$\mathrm{SE}_D = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}},$$

and the margin of error for confidence C is $m = z^* SE_D$, where z^* is the value of the $N(0,1)$ RV with area C between $-z^*$ and z^*. The approximate level C confidence interval for p is

$$D \pm m.$$

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$$\tilde{D} = \tilde{p}_1 - \tilde{p}_2,$$

where $\tilde{p}_1 = \frac{X_1+1}{n_1+2}$ and $\tilde{p}_2 = \frac{X_2+1}{n_2+2}$.

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Basically, we're just adding two observations to each sample, assuming that one was a success and one was a failure. In order to use the plus four confidence interval method, we have to have that the sample sizes are at least five for each sample.

§8.2 Comparing Two Proportions

Example (pass rates)

Students study for the same test in two different ways. One group of students studies individually. Another studies in groups of 5. A SRS of 100 independent-study students showed that 85 students passed. A SRS of 120 group-study students showed that 105 students passed. Create a 85% confidence interval for the difference of the proportion of students who passed in the independent-study population and the proportion of students who passed in the group-study population.

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Answer: Well, $D = -0.025$.

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Example (arrive on time)

A commuter is interested in which of two routes will help her arrive on time to work most frequently. She drives Route A 10 days and arrives on time 9 times. She drives Route B 12 times and arrives on time 10 times.

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Answer: $\tilde{D} = 0.04762$. And $\text{SE}_{\tilde{D}} = 0.1536$. And $z^* = 1.96$. Thus, $m = 0.3011$, and our confidence interval is $(-0.2535, 0.3487)$.

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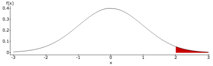
Definition (z significance test for comparing two proportions)

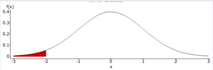
To test the hypothesis $H_0 : p_1 = p_2$ based on two SRSs of sizes n_1 & n_2 from populations with unknown proportions, compute the **test statistic**

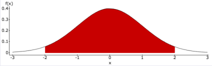
$$z = \frac{\hat{p}_1 - \hat{p}_2}{\mathrm{SE}_{Dp}}.$$

For a $N(0, 1)$ RV, the P-value for a test of H_0 against

$$\begin{aligned} H_a &: p_1 > p_2 \quad \text{is} \quad P(Z > z) \\ H_a &: p_1 < p_2 \quad \text{is} \quad P(Z < z) \\ H_a &: p_1 \neq p_2 \quad \text{is} \quad 2P(Z > |z|) \end{aligned}$$







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What's SE_{Dp}, you say? Is it the same standard error as before? No! It's not the same thing as SE_D. Because in the P-value test we're assuming under the null hypothesis that the two proportions are equal, we need to pool them! We calculate a new proportion by combining all successes and observations into a single proportion:

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We then use this proportion to calculate SE_{Dp}.

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$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2}.$$

We then use this proportion to calculate SE_{Dp}.

$$SE_{Dp} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}.$$

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- The P-values are based on the normal approximation to the binomial random variable, so the P-values are approximate.
- We need at least five successes and five failures in each of the samples.
- Always remember to use SE_{Dp} when you're assuming that the population proportions are equal in the null hypothesis.

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A note about comparing two population proportions in more generality: In general, your null hypothesis doesn't have to be that $p_1 = p_2$, which is equivalent to saying that $D = 0$. You could possible assume that $p_1 - p_2 = 0.15$, which is like saying that Population 1 has a proportion 15% higher than Population 2. Our method for P-values before would work fine, but we would no longer need to pool our proportions because we wouldn't be assuming that they are equal! So we'd use SE_D instead of SE_{Dp}.

Example (which drug is better)

Antibiotic A was given to 15 patients with a particular infection. After the treatment, 5 were still infected. Antibiotic B was given to 25 patients with the same infection. After treatment, 6 were still infected.

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Example (which drug is better)

Antibiotic A was given to 15 patients with a particular infection. After the treatment, 5 were still infected. Antibiotic B was given to 25 patients with the same infection. After treatment, 6 were still infected. Conduct a P-value test to test the claim that Antibiotic B is more effective than Antibiotic A. Use $\alpha = 0.20$.

Answer: Well, $\hat{p} = 0.725$.

Example (which drug is better)

Antibiotic A was given to 15 patients with a particular infection. After the treatment, 5 were still infected. Antibiotic B was given to 25 patients with the same infection. After treatment, 6 were still infected. Conduct a P-value test to test the claim that Antibiotic B is more effective than Antibiotic A. Use $\alpha = 0.20$.

Answer: Well, $\hat{p} = 0.725$. And $\mathrm{SE}_{Dp} = 0.14583$.

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Answer: Well, $\hat{p} = 0.725$. And $\mathrm{SE}_{Dp} = 0.14583$. And $z = -0.64$.

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Answer: Well, $\hat{p} = 0.725$. And $\text{SE}_{Dp} = 0.14583$. And $z = -0.64$. Thus, the P-value is 0.2611, and we fail to reject H_0 and do not support our claim.

Example (college graduates in cities)

Dalton wants to compare the proportion of college graduates in its city to that in Kingman. A SRS of 200 Dalton residents finds that 174 are college graduates while a SRS of 150 Kingman residents finds that 130 are college graduates. With $\alpha = 0.05$, test the claim that Dalton and Kingman have the same proportion of college graduates.

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Answer: Well, $\hat{p} = 0.86857$.

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Answer: Well, $\hat{p} = 0.86857$. And $\mathrm{SE}_{Dp} = 0.036494$.

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Answer: Well, $\hat{p} = 0.86857$. And $\text{SE}_{Dp} = 0.036494$. And $z = 0.09$.

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Answer: Well, $\hat{p} = 0.86857$. And $\text{SE}_{Dp} = 0.036494$. And $z = 0.09$. Thus, the P-value is 0.92722, and we fail to reject H_0 and support our claim.

Example (good eggs)

A new method of refrigeration is being considered to keep eggs fresher longer. 100 eggs were tested using the old method, and 90 were still good after 2 weeks. 120 eggs were tested using the new method, and 102 were good after two weeks. Use $\alpha = 0.15$ to test the claim that the old method is better than the new method.

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Answer: Well, $\hat{p} = 0.87273$. And $\mathrm{SE}_{Dp} = 0.045126$.

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Answer: Well, $\hat{p} = 0.87273$. And $SE_{Dp} = 0.045126$. And $z = 1.11$.

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Answer: Well, $\hat{p} = 0.87273$. And $SE_{Dp} = 0.045126$. And $z = 1.11$. Thus, the P-value is 0.1339, and we reject H_0 and support our claim.

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You may read about relative risk on page 520 in the text.