Lesson 2:
Chapter 2

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BSC Mathematics

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§2.1 Relationships

In this chapter, we will investigate relationships between two variables which describe the **same case**. For instance, height and weight can be two variables which are related if we are talking about the height and weight of the same case - like the height and weight of a horse. Variables can only be related if they both measure the same case!

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Is there an association between the balance on a loan and whether or not the loan is in default? If there is an association, is it weak or strong?

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In a study measuring the effects of light on attention, what is the explanatory variable? What is the response variable? (Assume that attention is measured by the time in which a subject can complete a menial task.)

§2.2 Scatterplots

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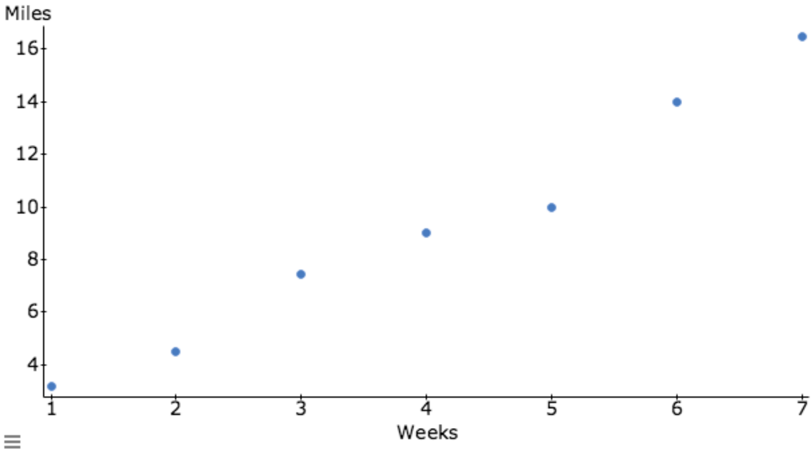
You can see positive/negative associations in scatterplots. Positive associations are those where the variables increase together. Negative associations are those where one variable increases as the other decreases. You can also see the **strength** of an association from the scatterplot.

§2.2 Scatterplots



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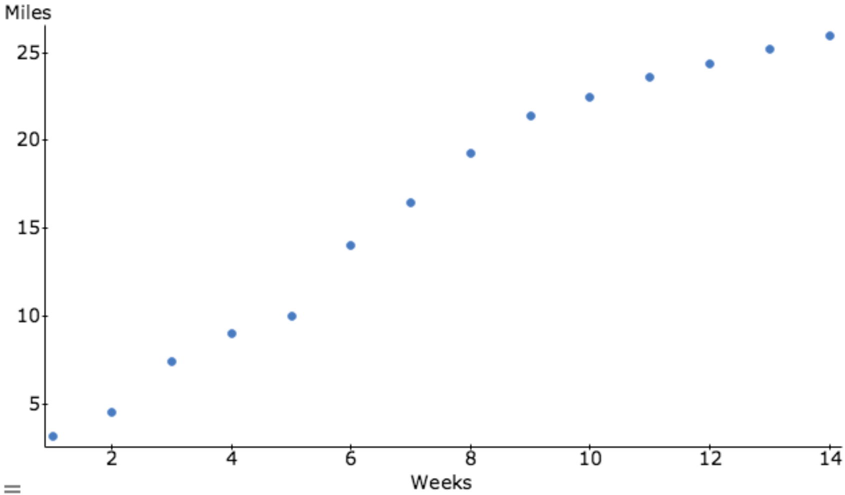

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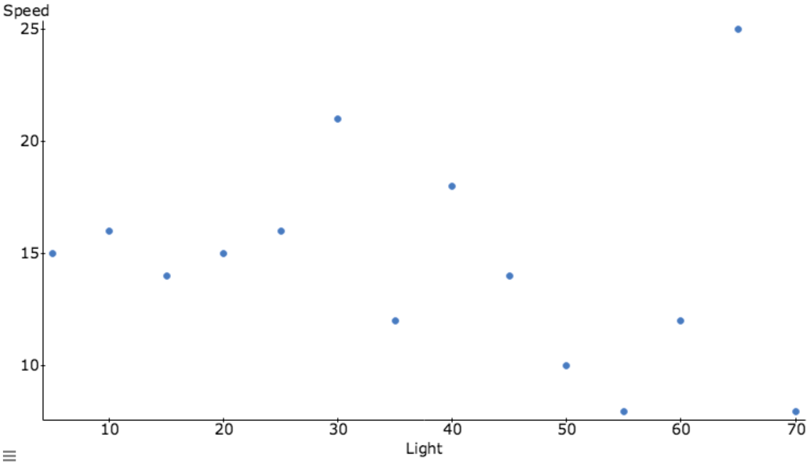

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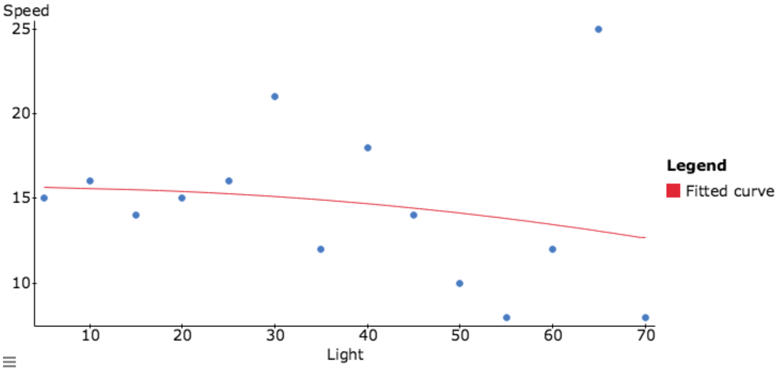

§2.3 Correlation

Definition (correlation)

The correlation between two variables is given by

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

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Here's the data for the miles ran after a certain number of weeks of training:

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Example

It's easy to check that $\bar{x} = 4$, $s_x = 2.160$, $\bar{y} = 9.239$, and $s_y = 4.798$. So, we calculate r:

$$\frac{1}{6}\left(\left(\frac{1-4}{2.160}\right)\left(\frac{3.2-9.239}{4.798}\right)+\cdots+\left(\frac{7-4}{2.160}\right)\left(\frac{16.5-9.239}{4.798}\right)\right).$$

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This gives $r = 0.988$.

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1. The sign of r indicates the nature of the relationship.
2. Correlation can only be calculated for **linear** relationships. Non-linear relationships can be identified by their scatterplot, and they should not have correlation calculated for them.
3. Correlation can only be calculated when we have matched pairs of data. Thus, there should always be the same number of x- and y-values.

§2.4 Least-Squares Regression

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Theorem

The least-squares regression line is always given by the formula $\hat{y} = b_0 + b_1 x$ *where* $b_1 = r\frac{s_y}{s_x}$ *and* $b_0 = \bar{y} - b_1\bar{x}$.

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We already calculated several pieces of summary data for the pairs below:

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- Causation and correlation are not the same thing!
- Two variables can be correlated linearly in a perfect fashion but not change by the same relative amounts.
- The r statistic is not resistant against outliers.