Lesson 4:
Chapter 4 Sections 1-2

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BSC Mathematics

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§4.1 Randomness

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Imagine drawing a card from a 52-card deck.

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A phenomenon is **random** if individual outcomes are uncertain (in some sense) but nonetheless display a regular distribution of outcomes when trails are repeated a large number of times.

Example (deck of cards)

Imagine drawing a card from a 52-card deck. What card would you draw? If you repeated this process 52 times, how often would you expect to have drawn an ace?

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What's the probability of drawing an ace from a 52-card deck?

You often can calculate the probability of an outcome or event if you make assumptions about the phenomenon rather than having to perform a long series of repetitions.

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1. Performing many trials can be difficult, so estimating probabilities or calculating them using mathematical assumptions is often necessary.
2. Trials must always be **independent**, i.e the outcomes of one trail must not depend on the outcome of any others.
3. Often we can imitate random behavior rather than actually performing the trials. We call this simulation.

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- gambling,
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- ergodicity/particle movement/brownian motion,
- finance, and much more.

Mastery Question:

Example (independence)

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- The number of holiday cards you receive over a period of ten years.
- The temperature at Vulcan on January 1st each year for three years.
- The results of five coin tosses where the coin is 40% likely to land on heads and 60% likely to land on tails.

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Mastery Question:

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- Whether or not a medical student passes her board exams.
- The numbers selected in a lottery drawing.

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- rolling a three
- rolling an odd number
- rolling an even number less than four

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Example (number of children)

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We summarize these facts in a table on the next slide.

Rule 1	$0 \leq P(A) \leq 1$
Rule 2	$P(S) = 1$
Rule 3	$P(A) + P(B) = P(A \text{ or } B)$ if A and B are disjoint
Rule 4	$P(A^c) = 1 - P(A)$

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It's important to understand how these identities arise, and looking at **Venn diagrams** can be useful for these and other identities.

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Example (hair color)

The table gives the proportion of hair colors in a class.

Black	Blond	Brown	Red	White/Grey	Other
0.31	0.11	0.28	0.08	0.04	0.18

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What's the probability that a student has a known hair color? 0.82. What's the probability that a student has either red or brown hair? 0.36.

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Example (rolling dice)

Create the probability model for rolling two dice.

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2	3	4	5	6	7
$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$

8	9	10	11	12
$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

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Theorem (equally likely outcomes)

If a random phenomenon has k possible outcomes, all equally likely, then each individual outcome has probability $\frac{1}{k}$. The probability of any event A is

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We didn't use this rule exactly because we "pinched together" certain outcomes since their result was the same in some sense, but the idea is the same.

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Definition (independence)

Two events A and B are **independent** if knowing that one occurs does not change the probability that the other occurs. If A and B are independent, then

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

We call this the multiplication rule for independent events.

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1. Drawing an ace from one deck of cards and then drawing a king from another deck.
2. Drawing an ace from one deck of cards and then drawing a king from the same deck without replacing the ace first.
3. Event A is disjoint from Event B. Are they independent?

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What's the probability that in a room of 25 people at least two people share a birthday? 56.87%.