Lesson 5:
Chapter 4 Section 3

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BSC Mathematics

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Example (equally likely events)

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- You spin a red in Twister or you spin a green.

§4.1-4.2 Review

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4. If $S = \{A, B\}$, then $P(A) = 0.5$ and $P(B) = 0.5$.
5. If $P(A)$ and $P(B)$ are both 0.4 and if $P(A \text{ and } B) = 0.8$, then A and B are independent.

§4.1-4.2 Review

Example (probability model)

Assume a die has been loaded so that 1 and 6 come up as often as each other and 2, 3, 4, 5 come put as often as each other. If $P(1 \text{ or } 6) = 0.52$, find the probabilities for each possible outcome.

Example (complicated event)

Only 5% of Australians have O-negative blood types. If 10 Australians are selected at random, what is the probability that at least one of them has O-negative blood?

§4.3 Random Variables

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Black	Brown	Blue	Green	Other
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If we chose 5 people at random, we could convert the eye-color variable into a numeric variable by recording the number of, for instance, brown eyes in the group of 5 people.

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If we chose 5 people at random, we could convert the eye-color variable into a numeric variable by recording the number of, for instance, brown eyes in the group of 5 people. Thus, the outcome

(Blue, Other, Brown, Black, Brown)

would be recorded as a 2.

§4.3 Random Variables

Definition (random variable)

A **random variable** is a variable whose value is a numeric outcome of a random process.

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Our random variable will look be as below.

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0.116	0.312	0.337	0.181	0.049	0.005

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This is a **binomial random variable** which we'll discuss in §5.

§4.3 Random Variables

Definition (discrete random variable)

A **discrete RV** X has possible values that can be given in an ordered list. The **probability distribution** of X lists the values of their probabilities.

Value of X	x_1	x_2	x_3	$\dots$
$P(X)$	p_1	p_2	p_3	$\dots$

The probabilities p_i must satisfy the following requirements.

1. $0 \leq p_i \leq 1$
2. $\sum p_i = 1$

You can find the probability of any event by adding up the probabilities p_i of the particular values of X that make up the event.

Is this a valid probability distribution?

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What is $P(X \leq 2)$? 0.765.

§4.3 Random Variables

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0	1	2	3	4	5
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What is $P(X \leq 2)$? 0.765. What's $P(1 < X \leq 3)$? 0.518.

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What does the histogram look like for our previous random variable?

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When a probability histogram is flat, you have a **uniform random variable**, i.e. a random variable where each value is equally likely!

What does the histogram look like for our previous random variable? What would it look like if it were uniform?

Uniform distributions could also be **continuous** rather than discrete!

§4.3 Random Variables

Uniform distributions could also be **continuous** rather than discrete!

Definition (continuous random variable)

A **continuous random variable** X takes all values in an interval (or possibly intervals) of numbers. The **probability distribution** of X is described by a density curve. The probability of any event is the area under the density curve and above the horizontal axis and between the values of X which bound the event.

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Note: Density curves must always be above the horizontal axis and the total area under them but above the horizontal axis must be 1.

§4.3 Random Variables

For a continuous random variable, $P(X = x) = 0$ for any value x, so we can ignore the distinction between $\leq$ and $<$ or between $\geq$ and $>$.

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A random variable X is uniform and continuous and takes values between 0 and 10. What is $P(1 < X \leq 4)$?

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Example (continuous uniform random variable)

A random variable X is uniform and continuous and takes values between 0 and 10. What is $P(1 < X \leq 4)$? $\frac{3}{10}$. Draw this area!

The most common continuous random variable that we will deal with is the normal random variable. The "interval" on which it takes values is the entire real line! How would you go about calculating the probability that a normal random variable takes values between a and b?

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$$z(a) = \frac{a-\mu}{\sigma} \quad \text{and} \quad z(b) = \frac{b-\mu}{\sigma}$$

§4.3 Random Variables

Standard Normal Probabilities

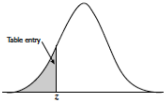

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

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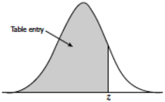

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0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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- the probability that X is between 1 and 9?

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- the probability that X is between 1 and 9? $P(1 < X < 9) = P(X < 9) - P(X < 1) = P(Z < 2) - P(Z < -2) =$

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- the probability that X is less than 7?
  $P(X < 7) = P(Z < 1) = 0.8413.$
- the probability that X is greater than 3? $P(X > 3) = 1 - P(X \leq 3) = 1 - P(Z \leq -1) = 1 - 0.1587 = 0.8413.$
- the probability that X is between 1 and 9?
  $P(1 < X < 9) = P(X < 9) - P(X < 1) = P(Z < 2) - P(Z < -2) = 0.9772 - 0.0228 = 0.9544.$

§4.3 Random Variables

There are ways of calculating standard normal probabilities in Minitab!