

Lesson 7: Chapter 5 Section 1

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BSC Mathematics

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§5.1 Sampling Distribution of a Sample Mean

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Example (sampling distribution)

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Example (sampling distribution)

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\bar{x}	4	4.25	4.5	4.75	5
$P(\bar{x})$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{5}{15}$	$\frac{3}{15}$	$\frac{2}{15}$

§5.1 Sampling Distribution of a Sample Mean

Example (sampling distribution)

Let's talk about the sampling distribution for the mean of a sample of size 2 for the following population. The population consists of 6 students with the following number of classes this term: 3, 4, 4, 5, 5, 6. Find the sampling distribution for sample size 2.

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\bar{x}	3.5	4	4.5	5	5.5
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Notice that the spread for this sampling distribution is **larger** than the spread for the previous sampling distribution! This is because the sample size was larger for the first example and smaller for the second. This is an example of the **Law of Large Numbers**.

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The **sampling distribution** of a statistic is the distribution of the random variable which arises from a randomized experiment or sample of fixed size.

They are indeed the same thing! In particular: **a statistic can be viewed as a random variable!**

§5.1 Sampling Distribution of a Sample Mean

Definition (population distribution)

A **population distribution** of a variable is the distribution of its values for all the members of the population. The population distribution is also the probability distribution of the variable when we choose one individual at random from the population.

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The population distribution is related to the sampling distribution, quite obviously. But it's also related to the way in which we collect data from the population! Ideally, we'll collect the data in a way that minimizes the dependence of the sampling distribution on the method of collection.

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- Sample means are more normal, i.e. they better mirror a normally distributed random variable, than individual observations.

What we really mean by these statements are that the **sampling distribution of the mean has a smaller variance and a more bell-shaped, symmetric distribution than the population distribution from which it is drawn.**

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Example (mean of the sampling distribution of the mean)

Calculate $\mu_{\bar{x}}$.

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Calculate $\mu_{\bar{x}}$. $\mu_{\bar{x}} = 4.5$.

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Example (mean of the sampling distribution of the mean)

Calculate $\mu_{\bar{x}}$. $\mu_{\bar{x}} = 4.5$. Notice $\mu_{\bar{x}} = \mu$.

Example (standard deviation of the sampling distribution of the mean)

Calculate $\sigma_{\bar{x}}$.

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Calculate $\sigma_{\bar{x}}$. $\sigma_{\bar{x}} = 0.3028$.

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Example (standard deviation of the sampling distribution of the mean)

Calculate $\sigma_{\bar{x}}$. $\sigma_{\bar{x}} = 0.3028$. Notice $\sigma_{\bar{x}} \neq \frac{\sigma}{\sqrt{n}}$. (But this is only because our samples were not independent, i.e. not done with replacement.)

§5.1 Sampling Distribution of a Sample Mean

We have the following theorem!

Theorem (mean and standard deviation of a sample mean)

Let \bar{x} be the mean of a SRS of size n from a population having mean μ and standard deviation σ . The mean and the standard deviation of the sample mean \bar{x} are

$$\mu_{\bar{x}} = \mu \quad \text{and} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}},$$

respectively.

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Theorem (sampling distribution of a sample mean drawn from a normal population)

If a population is normally distributed with mean μ and standard deviation σ , denoted $N(\mu, \sigma)$, then the sample mean \bar{x} of n independent observations has distribution $N(\mu, \frac{\sigma}{\sqrt{n}})$.

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If a population is normally distributed with mean μ and standard deviation σ , denoted $N(\mu, \sigma)$, then the sample mean \bar{x} of n independent observations has distribution $N(\mu, \frac{\sigma}{\sqrt{n}})$.

Theorem (Central Limit Theorem)

*If a SRS of large size n is drawn from **ANY** population with mean μ and standard deviation σ , then the sample mean \bar{x} has approximately a normal distribution $N(\mu, \frac{\sigma}{\sqrt{n}})$.*

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Basically, in order to say that the sampling distribution of the sample mean is (roughly)

$$N\left(\mu, \frac{\sigma}{\sqrt{n}}\right),$$

one of the two following criteria must be met:

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one of the two following criteria must be met:

- 1 The population from which the sample is drawn must be itself normally distributed. **OR**
- 2 The SRS size n must be large. (For now, let's say $n \geq 30$.)

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Example (How is the sample mean distributed?)

You take a SRS of 36 apples from an orchard whose apples are 7 ounces on average with a variation of 4 ounces squared. How is \bar{x} distributed?

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$$N\left(7, \frac{\sqrt{4}}{\sqrt{36}}\right) = N\left(7, \frac{1}{3}\right).$$

If the average weight of these 36 apples was 9 ounces, would you be surprised?

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If the average weight of these 36 apples was 9 ounces, would you be surprised? If the weight of a single apple was 9 ounces, would you be surprised?

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Example (Which point estimate for the mean is more unusual?)

Test scores in a class of 200 students are normally distributed. The mean and standard deviation for test scores is 75 and 10, respectively. A SRS is taken of 25 scores and \bar{x}_1 is 73. Another SRS of 4 scores is taken and \bar{x}_2 is 85. Which observation is more unusual, \bar{x}_1 or \bar{x}_2 ?

§5.1 Sampling Distribution of a Sample Mean

Example (Don't insure only a few people.)

Exotic insurance policies are often expensive. Let's see why! Assume that individual losses on some insurance product are $N(12000, 1000)$. What's the probability that, if you insure 12 people at the cost of \$12200 each, you'll lose money? What's the probability that, if you insure 1200 people at the cost of \$12200 each, you'll lose money?

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$$P(\bar{X}_{12} > 12200) = P(Z > 0.69) = 0.2451.$$

$$P(\bar{X}_{1200} > 12200) = P(Z > 6.928) \approx 0.$$

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$$P(\bar{X}_{12} > 12200) = P(Z > 0.69) = 0.2451.$$

$$P(\bar{X}_{1200} > 12200) = P(Z > 6.928) \approx 0.$$

So, obviously, there is no risk of losing money when you sell many policies. The least risky types of insurance (from the insurer's perspective) are policies that are sold to **many** people.

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Example (getting a better estimate)

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So! We need to take a sample of size more than 400!

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So! We need to take a sample of size more than 400!

What might you need to do if the n you got in the above example were smaller than 30?

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Theorem (linear combinations of independent normal RVs are also normal)

If X and Y are independent normal random variables, then so is $aX + bY$, where a and b are constants.

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Therefore, the standard deviation of $aX + bY$ is

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$$\sigma_{aX+bY}^2 = a^2\sigma_X^2 + b^2\sigma_Y^2.$$

Therefore, the standard deviation of $aX + bY$ is $\sqrt{a^2\sigma_X^2 + b^2\sigma_Y^2}$.

§5.1 Sampling Distribution of a Sample Mean

Example (class averages)

Class E has an average on a final exam of 75 with standard deviation of 15. Class B has an average of 78 on the final exam with standard deviation of 20. What's the probability that if we randomly sample 35 grades from Class E and 40 grades from Class B that the average of the sample from Class E will be greater than the average of the sample from Class B ? Assume that these sample averages are independent.

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The sample from Class E is $N(75, 2.535)$, and the sample from Class B is $N(78, 3.162)$. We can calculate

$$P(\bar{X}_E > \bar{X}_B) =$$

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The sample from Class E is $N(75, 2.535)$, and the sample from Class B is $N(78, 3.162)$. We can calculate

$$P(\bar{X}_E > \bar{X}_B) = P(\bar{X}_B - \bar{X}_E < 0).$$

But we know that the difference of the means is $N(3, 4.053)$, so $P(\bar{X}_B - \bar{X}_E < 0) = P(Z < -0.74) = 0.2296$.

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Some extra notes about the Central Limit Theorem

- A more general version of the CLT says that distribution of the sum or average of many small random quantities is nearly normal, regardless of whether or not they come from the same distribution or are independent.

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- The CLT applies to discrete random variables as well as continuous random variables!

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Some extra notes about the Central Limit Theorem

- A more general version of the CLT says that distribution of the sum or average of many small random quantities is nearly normal, regardless of whether or not they come from the same distribution or are independent.
- The CLT applies to discrete random variables as well as continuous random variables!
- There are CLT-like statements for non-normal random variables as well. For instance, the sampling distributions of sample means for **exponential RVs** is often modeled by a **Weibull** distribution. These model things like time-to-failure of electrical components, mortality, and waiting times.