Lesson 9:
Chapter 6 Sections 1-2

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BSC Mathematics

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§6.1 Estimating with Confidence

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Descriptive statistics involve producing summary statistics of data sets.

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Definition (inferential statistics)

Inferential statistics is the process of drawing conclusions about populations based on the descriptive/summary statistics calculated from a sample of that population.

§6.1 Estimating with Confidence

Our first step in drawing conclusions about a population involves drawing conclusions about how well a statistic mirrors its population parameter. Think about this example.

Example (Samples of 100 SAT scores will produce intervals containing μ 95% of the time.)

Suppose that you took a sample of 100 students' SAT scores and found that the mean of this sample was

$$\bar{x} = 1450.$$

Let's say we also know that SAT scores have a standard deviation of $\sigma = 120$.

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So, if we take SRSs of size 100 from this population a large number of times, 95% of those samples will produce intervals of the form $(\bar{x} - 2\sigma_{\bar{x}}, \bar{x} + 2\sigma_{\bar{x}})$ which contain μ.

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So, if we take SRSs of size 100 from this population a large number of times, 95% of those samples will produce intervals of the form $(\bar{x} - 2\sigma_{\bar{x}}, \bar{x} + 2\sigma_{\bar{x}})$ which contain μ. **This is what we mean by confidence in a "confidence interval!"**

§6.1 Estimating with Confidence

Definition (confidence interval)

A C-confidence interval for a parameter is an interval computed from a sample by a method that has probability C of producing an interval containing the true population parameter. Here, the probability is seen **from the perspective of the sampling distribution**!

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We will discuss methods of producing C-confidence intervals for means! The general form of these is

$$\left(\bar{x} - \frac{z^*\sigma}{\sqrt{n}}, \bar{x} + \frac{z^*\sigma}{\sqrt{n}}\right).$$

Here, z^* is the z-score corresponding to the confidence level C.

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5. Create your C-confidence interval $\bar{x} \pm m$.

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5. Create your C-confidence interval $\bar{x} \pm m$.

Note: The confidence level of this interval is exactly C when the population distribution is normal and approximately C when the sample size n is large.

Example (creating a confidence interval)

Calories in a meal deal at a local restaurant are normally distributed with $\sigma = 10$. A sample of 10 meals was tested for caloric value, and the following dataset resulted.

$$450, 455, 475, 477, 477, 480, 481, 490, 495, 495$$

Create a 90% confidence interval for μ.

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Example (creating a confidence interval)

Calories in a meal deal at a local restaurant are normally distributed with $\sigma = 10$. A sample of 10 meals was tested for caloric value, and the following dataset resulted.

$$450, 455, 475, 477, 477, 480, 481, 490, 495, 495$$

Create a 90% confidence interval for μ.

Answer: (472.3,482.7).

Example (how sample size affects confidence intervals)

Let's say we had a sample of size 100 and a sample of size 10 from the same population as the previous example, and let's say that the means of both of these samples was 477.5. What's the two confidence intervals?

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Example (how sample size affects confidence intervals)

Let's say we had a sample of size 100 and a sample of size 10 from the same population as the previous example, and let's say that the means of both of these samples was 477.5. What's the two confidence intervals?

Answer: (472.3,482.7) for sample size 10 and (475.9,479.1) for sample size 100.

Note: Larger sample sizes create narrower confidence intervals.

§6.1 Estimating with Confidence

Example (how confidence levels affect confidence intervals)

Let's say we had a sample of size 10 from the same population as before, and let's say that the means of both of these samples was 477.5. What is the 90%-confidence interval? What is the 95% confidence interval?

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Example (how confidence levels affect confidence intervals)

Let's say we had a sample of size 10 from the same population as before, and let's say that the means of both of these samples was 477.5. What is the 90%-confidence interval? What is the 95% confidence interval?

Answer: (472.3,482.7) for 90% confidence and (471.3,483.7) for 95% confidence.

Note: Higher confidence makes confidence intervals wider!

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1. Find a way to reduce σ — but this option is almost always impossible.
2. Decrease confidence C — but this results in a less confident result, even though it's easy to do.
3. Increase n — and this maintains confidence, even though it requires a little extra work.

§6.1 Estimating with Confidence

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$$m = z^* \frac{\sigma}{\sqrt{n}} \implies n = \left(\frac{\sigma \cdot z^*}{m} \right)^2$$

Use the second identity and **always round up** to get the n needed for the desired m.

Example (how to choose n for a given m)

Say you have a population whose standard deviation is 16. You want to produce an 80% confidence interval for the mean of this population, but you'd like to ensure that the margin of error is no more than 2. How big should your sample be?

Example (how to choose n for a given m)

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Answer: n should be at least 106.

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Pages 365-366 contain warnings about inference that you should look at carefully on your own.

Here's some motivation for this topic.

Example (significance testing)

Say you believed your friend to be an honest person and not a liar/cheater.

§6.2 Tests of Significance

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Example (significance testing)

Say you believed your friend to be an honest person and not a liar/cheater. If you played a hand of poker and she won, would you change your mind? What if she won two hands in a row? Three? 10,000? If she consistently won for a long time, you might have to abandon your assumption that your friend was an honest person — at least when it came to playing poker.

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A **significance test** is a method for determining when a result is statistically significant enough to reject an assumption.

The assumption we make is usually called the **null hypothesis**. But sometimes we may want to test a claim which is better formulated negatively, and we sometimes use the negative statement as the null hypothesis.

§6.2 Tests of Significance

Definition (null hypothesis)

A **null hypothesis** H_0 is the statement being tested in a significance test. The test is designed to assess the strength of the evidence *against* the null hypothesis, i.e. in favor of the alternative hypothesis. The null hypothesis is usually a statement like "no affect" or "no difference" and thus contains equality in its formulation.

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Example (formulate the null hypotheses)

- Claim: The mean is no more than 5.

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Note: Null hypotheses tend to have equality. Alternative hypothesis never include equality and must always have one of the symbols $<$, $>$, or $\neq$. The first two symbols indicate that the test is one-sided. The last symbol indicates that the test is two-sided. A test is right-sided if $>$ is in the alternative hypothesis and left-sided if $<$ is in the alternative hypothesis.

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Test statistics are computed differently for different significance tests! It's important that you know what type of distribution your test statistic belongs to and how it's computed. Generally, they take the form

$$\text{test statistic} = \frac{\text{estimate} - \text{hypothesized value}}{\text{standard deviation of estimate}}.$$

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$$\text{test statistic} = \frac{\text{estimate} - \text{hypothesized value}}{\text{standard deviation of estimate}}.$$

Once this is computed, you can compute your P-value.

§6.2 Tests of Significance

Definition (P-value)

A P-**value** is the probability, assuming H_0 is true, that the test statistic would take a value as extreme or more extreme than that actually observed. The smaller the P-value, the greater the evidence against H_0 provided by the data.

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P-values relate to confidence/significance in the following way:

$$\frac{C}{100} = 1 - \alpha,$$

where α is your significance level. You **reject** H_0 when the P-value is less than or equal to α because the test statistic/data are then statistically significant at level α. If the P-value is greater than α, then the data is not statistically significant enough to reject H_0, so you fail to reject it.

§6.2 Tests of Significance

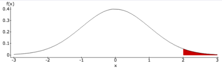
Definition (z test for population mean)

To test the hypothesis $H_0 : \mu = \mu_0$ based on a SRS of size n from a population with unknown μ but known σ, compute the **test statistic**

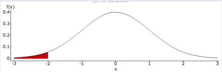
$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}.$$

For a $N(0, 1)$ RV, the P-value for a test of H_0 against

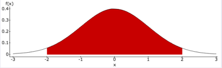
$H_a : \mu > \mu_0$ is $P(Z > z)$



$H_a : \mu < \mu_0$ is $P(Z < z)$



$H_a : \mu \neq \mu_0$ is $2P(Z > |z|)$



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So, in order to use the previous test, σ must be known, a SRS must be taken, and **either** the population must be normally distributed or the sample size n must be large.

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So, in order to use the previous test, σ must be known, a SRS must be taken, and **either** the population must be normally distributed or the sample size n must be large. Let's do an example!

§6.2 Tests of Significance

Example (hypothesis test)

The mean calories in a meal deal were 475, but the restaurant has changed the meal to be lighter. You want to test the claim that the new meal deal has fewer calories. Assume the calories are normally distributed and that $\sigma = 5$ calories. You want to test with 5% significance. You get the following sample:

$$475, 455, 470, 477, 489, 435, 445, 447, 475, 476.$$

Does this data support your claim?

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Does this data support your claim?

Answer: P-value is essentially 0. Reject H_0, and support H_a or your claim!

Example (hypothesis test)

An aspirin bottle's label indicates that it contains 325mg of aspirin. You want to test this claim. The standard deviation of the amount of aspirin in these pills is $\sigma = 2$mg. You collect a sample of 100 pills and find that the mean amount of aspirin in these pills is 324.6mg. With a 10% significance, test the claim.

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Answer: P-value is 0.0455. Reject H_0, and support H_a. Do not support your claim.

Example (hypothesis test)

A company's union insists that average pay of its employees is at least \$16 per hour. A random sample of 32 employees is taken, and their average pay is computed to be \$15.90 per hour. Test the claim that average pay is at least \$16 per hour assuming that the standard deviation for pay is \$1.50 per hour. Use $\alpha = 0.01$.

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Answer: P-value is 0.353. Fail to reject H_0, and do not support H_a. Support your claim.

You can also test claims using confidence intervals.

Definition (two-sided significance tests using confidence intervals)

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For one-sided tests using confidence intervals, you must **double your significance** to compute the corresponding confidence.

Example (hypothesis test using confidence interval)

Say you create a 90% confidence interval to test the claim that a mean is greater than 10. The resulting confidence interval is $(9.9, 10.2)$. Do you support or fail to support your claim? What's the significance of this test?

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Answer: Do not reject H_0 and fail to support your claim. The significance is 5%.

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Example (hypothesis test using a critical value)

You want to test the claim that a city's average home price is \$175000. The standard deviation of home prices in this city is $\sigma = \$25000$. You take a SRS of 40 homes and find that their mean price is \$168000. Use a critical value significance test with $\alpha = 0.02$ to test this claim.

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Answer: The test statistic is $z = -1.77$. The critical value is 2.33. Thus, do not reject H_0 because $|-1.77| < 2.33$. Support your claim.